1) Tracciare il grafico della funzione

$$f(x) = e^{-x}(x-1)^2$$

a) <u>Dominio</u> $A = \mathbb{R}$

b) <u>Pari o dispari</u>

$f(-x) = e^{x}(-x-1)^2 \neq f(x)$ la f non è pari

$-f(x) = -e^{-x}(x-1)^2 \neq -f(x)$

la f non è dispari

c) <u>Int. assi</u>

$$\begin{cases} y = e^{-x}(x-1)^2 \\ x = 0 \end{cases} \qquad \begin{cases} y = 1 \\ x = 0 \end{cases}$$

$P(0,1)$

$$\begin{cases} y = e^{-x}(x-1)^2 \\ y = 0 \end{cases}$$

$$\begin{cases} e^{-x}(x-1)^2 = 0 \\ y = 0 \end{cases}$$

$e^{-x} = 0 \quad \nexists\, x \in \mathbb{R}$

$(x-1)^2 = 0 \quad x = 1$

$Q(1, 0)$

d) <u>Segno</u>

$e^{-x}(x-1)^2 > 0$ I F $e^{-x} > 0 \quad \forall x \in \mathbb{R}$

II F $(x-1)^2 > 0 \quad \forall x \in \mathbb{R} - \{1\}$

		1	
I F	+ + +		+ + +
II F	+ + +	0	+ +
	+	0	+

e) Asintoti $(x \to +\infty\ ,\ x \to -\infty)$

$$\lim_{x \to +\infty} e^{-x}(x-1)^2 = [0 \cdot \infty]\ FI$$

$$\lim_{x \to +\infty} \frac{(x-1)^2}{e^x} = \left[\frac{\infty}{\infty}\right] FI \qquad \boxed{e^{-x} = \frac{1}{e^x}}$$

$$\lim_{x \to +\infty} \frac{2 \cdot (x-1) \cdot 1}{e^x} = \lim_{x \to +\infty} \frac{2(x-1)}{e^x} = \left[\frac{\infty}{\infty}\right] FI$$

$$\lim_{x \to +\infty} \frac{2 \cdot 1}{e^x} = \frac{2}{+\infty} = 0$$

$y=0$ as. oriz destro

$$\lim_{x \to -\infty} e^{-x}(x-1)^2 = +\infty \quad \nexists \text{ as. oriz. sx}$$

ma è verificata la C.N. per l'esistenza dell'as. obliquo sx

$$\lim_{x \to -\infty} \frac{e^{-x}(x-1)^2}{x} = \left[\frac{\infty}{\infty}\right] FI$$

$$\lim_{x \to -\infty} \frac{e^{-x}\left(x\left(1-\frac{1}{x}\right)\right)^2}{x} = \qquad \frac{1}{x} \to 0$$

$$\lim_{x \to -\infty} \frac{e^{-x} \cdot x^{\cancel{2}}}{\cancel{x}} = \lim_{x \to -\infty} e^{-x} \cdot x = -\infty$$

$\nexists$ as. obliquo sinistro

f) <u>Max e min</u>

$$f(x) = e^{-x}(x-1)^2$$

$$f'(x) = \underbrace{e^{-x}(-1)(x-1)^2}+\underbrace{e^{-x} \cdot 2(x-1) \cdot 1}$$

$$f'(x) = e^{-x}(x-1)\left[-1(x-1)+2\right]$$

$$f'(x) = e^{-x}(x-1)(-x+1+2)$$

$$f'(x) = e^{-x}(x-1)(3-x)$$

C.N. $f'(x) = 0$ $\qquad e^{-x} = 0 \;\; \nexists x$

$$e^{-x}(x-1)(3-x) = 0 \quad \begin{cases} x-1=0 & x=1 \\ 3-x=0 & x=3 \end{cases}$$

$x=1$ e $x=3$ punti critici

C.S $f'(x)>0$

$e^{-x}(x-1)(3-x)>0$

I F $e^{-x}>0 \quad \forall x \in \mathbb{R}$

II F $x-1>0 \qquad x>1$

III F $3-x>0$

$-x>-3 \qquad x<3$

		1		3	
I F	+ +		+		+ + + +
II F	- - -	0	+		+ + + +
III F	+ + + + +			0	- - - -
y'	$-$	0	$+$	0	$-$
	↘		↗		↘

$x=1 \qquad y=0 \qquad (1,0)$ min rel.

$x=3 \qquad y=e^{-3}\cdot 4 = 4e^{-3}$

$(3\,;\,4e^{-3})$ max rel

g) Flem

$f'(x) = e^{-x}(x-1)(3-x)$

$f'(x) = e^{-x}(3x - x^2 - 3 + x)$

$f'(x) = e^{-x}(-x^2 + 4x - 3)$

$f''(x) = e^{-x}(-1)(-x^2+4x-3) + e^{-x}(-2x+4)$

$= e^{-x}(x^2 - 4x + 3 - 2x + 4)$

$= e^{-x}(x^2 - 6x + 7)$

C. N $\quad f''(x) = 0$

$e^{-x}(x^2-6x+7) = 0$

$e^{-x} = 0 \quad \nexists x$

$x^2 - 6x + 7 = 0$

$$x_{1/2} = \frac{6 \pm \sqrt{36-28}}{2}$$

$$x_{12} = \frac{6 \pm \sqrt{8}}{2} = \frac{6 \pm 2\sqrt{2}}{2} = \frac{\cancel{2}(3 \pm \sqrt{2})}{\cancel{2}}$$

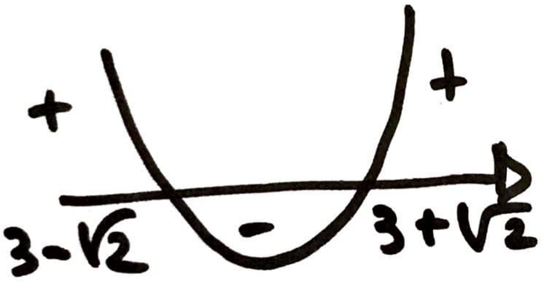
$x = 3 \pm \sqrt{2}$ probabile punti di flesso

C.S $f''(x) > 0$

$e^{-x}(x^2 - 6x + 7) > 0$

I F $e^{-x} > 0 \quad \forall x \in \mathbb{R}$

II F $x^2 - 6x + 7 > 0$



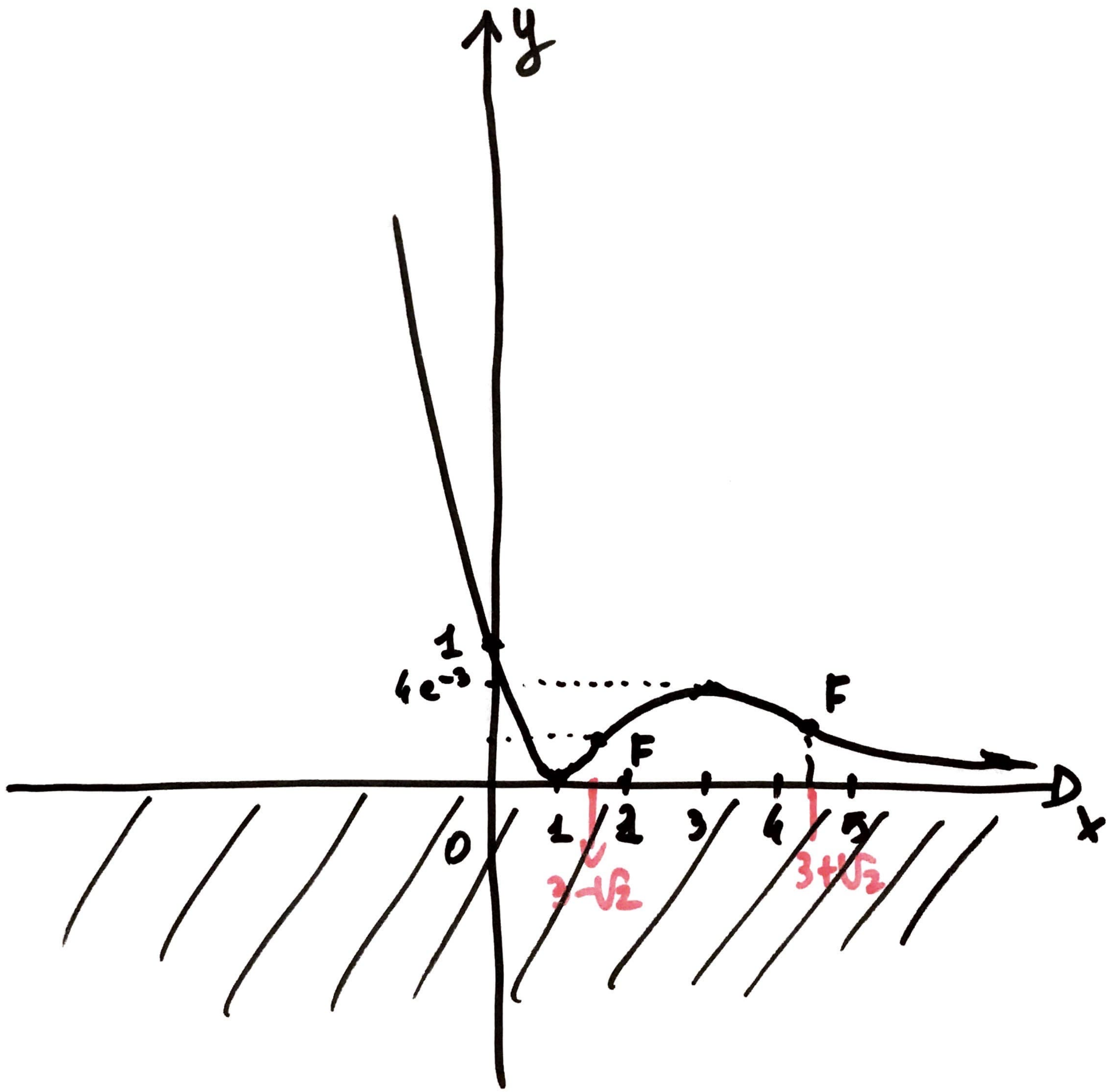
$x < 3-\sqrt{2} \vee x > 3+\sqrt{2}$

		$3-\sqrt{2}$		$3+\sqrt{2}$	
I F	+ + +		+ +		+ +
II F	+ +	0	−	0	+ +
y''	+	0	−	0	+
	∪	F	∩	F	∪

$x = 3-\sqrt{2}$ flesso $\quad f(3-\sqrt{2})$

$(3-\sqrt{2} \,;\, f(3-\sqrt{2}))$ flesso

$\left(3+\sqrt{2}\ ;\ f(3+\sqrt{2})\right)$ flesso



y
x
1
4e^{-3}
0
1
2
3
4
5
F
F
3-√2
3+√2

2) Det. per via grafica il dominio della f.

$$y = \sqrt{-\ln(|x|) - e^{|x-1|}}$$

$$\begin{cases} |x| > 0 \text{ è compresa II} \\ -\ln(|x|) - e^{|x-1|} \geq 0 \end{cases}$$

$$-\ln(|x|) \geq e^{|x-1|}$$

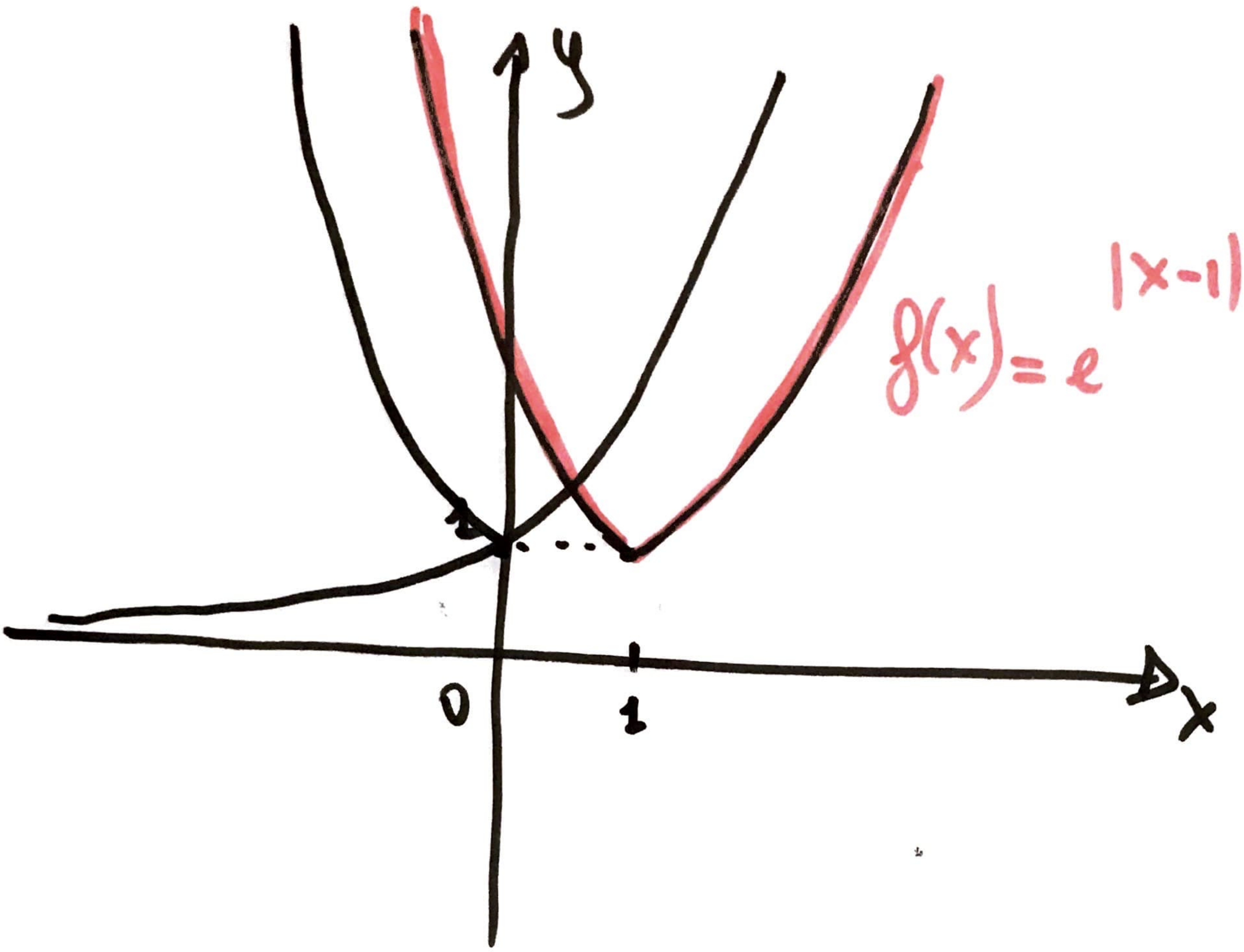
$$f(x) = -\ln(|x|) \qquad g(x) = e^{|x-1|}$$

$$f(x) \geq g(x)$$

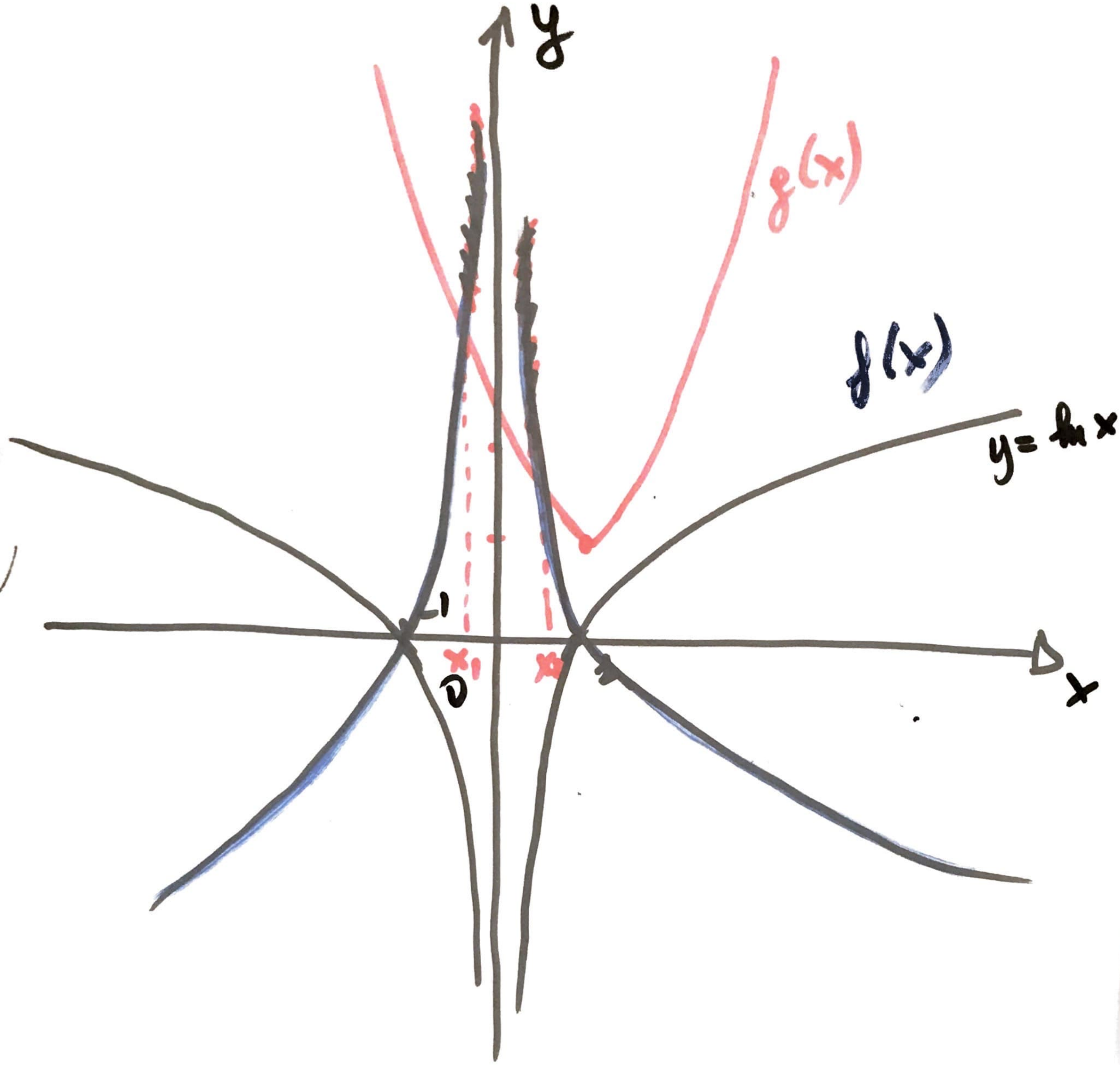
$$y = \ln x \xrightarrow[f(|x|)]{} y = \ln(|x|) \xrightarrow[-f(x)]{} y = -\ln(|x|)$$

$$y = e^x \xrightarrow[f(|x|)]{} y = e^{|x|} \xrightarrow[f(x-k)]{} y = e^{|x-1|}$$

$$k = 1$$



y
$f(x) = e^{|x-1|}$
1
0
1
x



$f(x) = -\ln(|x|)$

$D: x_1 \leq x < 0 \vee 0 < x \leq x_2$

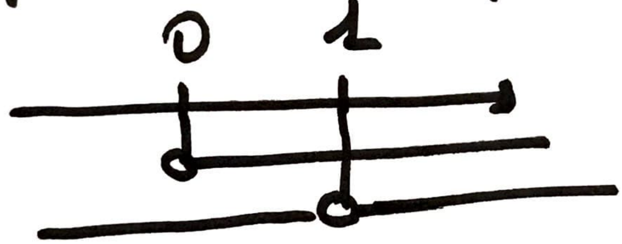
$D = [x_1, 0) \cup (0, x_2]$

3) Determinare gli asintoti della funzione

$$f(x) = x+2+\frac{2}{\ln x}$$

Dominio

$$\begin{cases} x>0 \\ \ln x \neq 0 \to e^{\ln x} \neq e^{0} \qquad x \neq 1 \end{cases}$$



$$D = (0,1) \cup (1,+\infty)$$

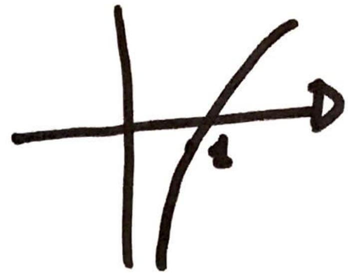
$$(x \to 0^+,\ x \to 1^-,\ x \to 1^+,\ x \to +\infty)$$

$$\lim_{x \to 0^+} \left(x+2+\frac{2}{\ln x} \right) = 0+2+\frac{2}{-\infty}$$

$$= 2 \qquad \nexists \text{ as.}$$

$$\lim_{x \to 1^-} \left(x + 2 + \frac{2}{\ln x} \right) =$$

$$1 + 2 + \frac{2}{0^-}$$



$$= -\infty$$

$$\lim_{x \to 1^+} \left(x + 2 + \frac{2}{\ln x} \right) = 1 + 2 + \frac{2}{0^+} = +\infty$$

$x = 1$ as. verticale per la f.

$$\lim_{x \to +\infty} \left(x + 2 + \frac{2}{\ln x} \right) = +\infty + 2 + \frac{2}{+\infty}$$

$$= +\infty$$

∄ as. orizz. destro ma è soddisfatta la C.N. per esistenza asintoto obliquo destro

$$\lim_{x \to +\infty} \left(x + 2 + \frac{2}{\ln x} \right) \cdot \frac{1}{x} =$$

$$\lim_{x \to +\infty} \left(1 + \underbrace{\frac{2}{x}}_{\to 0} + \underbrace{\frac{2}{x \ln x}}_{\to 0} \right) = 1 \ (m)$$

$$\lim_{x\to+\infty}\left[f(x)-mx\right]$$

$$\lim_{x\to+\infty}\left(\cancel{x}+2+\frac{2}{\ln x}-\cancel{x}\right)=$$

$$\lim_{x\to+\infty}\left(2+\overbrace{\frac{2}{\ln x}}^{\to 0}\right)=2 \quad (q)$$

$$y = mx + q$$

$y = x + 2$ as. obliquo destro per la f.

4) Det. per quale valore reale positivo del parametro K si ha

$$\lim_{x \to -\infty} \frac{-2x+7}{\sqrt{Kx^2+3x-1}} = 4$$

$$\lim_{x \to -\infty} \frac{-2x+7}{\sqrt{Kx^2+3x-1}} = \left[\frac{+\infty}{+\infty}\right] \text{ FI}$$

$$\lim_{x \to -\infty} \frac{x\left(-2+\underbrace{\frac{7}{x}}_{\to 0}\right)}{\sqrt{x^2\left(K+\underbrace{\frac{3}{x}}_{\to 0}-\underbrace{\frac{1}{x^2}}_{\to 0}\right)}} =$$

$$= \lim_{x \to -\infty} \frac{-2x}{\sqrt{Kx^2}} = \lim_{x \to -\infty} \frac{-2x}{|x|\sqrt{K}}$$

$$= \lim_{x \to -\infty} \frac{-2\cancel{x}}{\cancel{-x}\sqrt{K}} = \frac{2}{\sqrt{K}}$$

Cond. $\frac{2}{\sqrt{K}} = 4 \qquad 2 = 4\sqrt{K}$

$\sqrt{K} = \frac{1}{2}$ $\qquad K = \frac{1}{4}$

5) $\lim_{x \to 0} \left(1 - 5x^2\right)^{\frac{3K}{2x^2}} = 3 \quad K = ?$

$$\lim_{x \to 0} \left(1 - 5x^2\right)^{\frac{3K}{2x^2}} = 1^{\infty} \text{ FI}$$

$$\lim_{x \to \alpha} \left(1 + \frac{1}{f(x)}\right)^{f(x)} = e \qquad \lim_{x \to \alpha} f(x) = \pm\infty$$

$$\lim_{x \to 0} \left[\left(1 + \frac{1}{-\frac{1}{5x^2}}\right)^{-\frac{1}{5x^2}}\right]^{-5x^2 \cdot \frac{3K}{2x^2}}$$

$$\lim_{x \to 0} -\frac{1}{5x^2} = -\infty$$

$$= \lim_{x \to 0} e^{-5\cancel{x^2} \cdot \frac{3k}{2\cancel{x^2}}} = e^{-\frac{15K}{2}}$$

cond.

$$e^{-\frac{15k}{2}} = 3$$

$$\ln e^{-\frac{15k}{2}} = \ln 3$$

$$\frac{-15k}{2} = \ln 3$$

$$-15k = 2\ln 3$$

$$\boxed{k = -\frac{2\ln 3}{15}}$$

6) Det $a, b \in \mathbb{R}$ in modo che la funzione

$$f(x) = \begin{cases} a\sqrt{x^2+4} + b\ln(x^2+1) + x & x \geq 0 \\ \dfrac{b e^{2x}}{x-1} & x < 0 \end{cases}$$

sia cont. e derivabile in $x = 0$

$\left[y = \dfrac{b e^{2x}}{x-1} \quad x \neq 1 \quad (x<0) \right.$
la f è def. nell'intorno di riferimento $\left. \right]$

Continuità in $x = 0$

$$\lim_{x \to 0^+} \left(a\sqrt{x^2+4} + b\ln(x^2+1) + x \right) =$$

$$= 2a = f(0)$$

$$\lim_{x \to 0^-} \frac{b e^{2x}}{x-1} = \frac{b}{-1} = -b$$

$$\boxed{2a = -b}$$

<u>Derivabilità in $x_0 = 0$</u>

$$f'(x) = \begin{cases} a \dfrac{1}{\cancel{2}\sqrt{x^2+4}} \cdot \cancel{2}x + b \dfrac{1}{x^2+1} 2x + 1 & x > 0 \\[2ex] \dfrac{b e^{2x} \cdot 2 (x-1) - b e^{2x} \cdot 1}{(x-1)^2} & x < 0 \end{cases}$$

$$= \frac{e^{2x} (2bx - 2b - b)}{(x-1)^2}$$

$$f'(x) = \begin{cases} \dfrac{a x}{\sqrt{x^2+4}} + \dfrac{2bx}{x^2+1} + 1 & x > 0 \\[2ex] \dfrac{e^{2x}(2bx - 3b)}{(x-1)^2} & x < 0 \end{cases}$$

$$\lim_{x\to 0^+}\left(\frac{a x}{\sqrt{x^2+4}}+\frac{2bx}{x^2+1}+1\right)=1$$

$$\lim_{x\to 0^-}\left(\frac{e^{2x}(2bx-3b)}{(x-1)^2}\right)=-3b$$

$$\boxed{-3b=1}$$

$$\begin{cases}2a=-b\\-3b=1\end{cases}\qquad\begin{cases}2a=\frac{1}{3}\\b=-\frac{1}{3}\end{cases}$$

$$\begin{cases}6a=1\\b=-\frac{1}{3}\end{cases}\qquad a=\frac{1}{6}$$

Per $a=\frac{1}{6}$ e $b=-\frac{1}{3}$ la

$f(x)$ è contr. e der. in

$x=0$

7) Tracciare il grafico della funzione

$$f(x) = \frac{4x-4}{x^3}$$

a) <u>Dominio</u> $x^3 \neq 0$ $x \neq 0$

$D = \mathbb{R} - \{0\}$

b) <u>Pari o dispari</u>

$f(-x) = \frac{-4x-4}{-x^3} = \frac{4x+4}{x^3} \neq f(x)$ non è pari

$-f(x) = \frac{-4x+4}{x^3} \neq f(-x)$ non è dispari

c) <u>Int. assi</u>

$\begin{cases} y = \frac{4x-4}{x^3} \\ y = 0 \end{cases}$ $\begin{cases} \frac{4x-4}{x^3} = 0 \\ y = 0 \end{cases}$ $4x-4=0$
$x = 1$

$P(1, 0)$

d) <u>Segno</u>

$$\frac{4x-4}{x^3} > 0$$

N $4x-4>0$

$x>1$

D $x^3>0$

$x>0$

		0		1	
N	$-$	$-$	$-$	0	$+$ $+$ $+$
D	$-$ $-$	0	$+$	$+$	$+$ $+$
	$+$	$\nexists$	$-$	0	$+$

e) <u>Asintoti</u> $(x \to 0^+,\ x \to 0^-,\ x \to +\infty,\ x \to -\infty)$

$$\left.\begin{aligned} &\lim_{x \to 0^+} \frac{4x-4}{x^3} = \frac{-4}{0^+} = -\infty \\ &\lim_{x \to 0^-} \frac{4x-4}{x^3} = \frac{-4}{0^-} = +\infty \end{aligned}\right\} \ x=0 \text{ as. vert.}$$

$$\lim_{x \to +\infty} \frac{4x-4}{x^3} = \left[\frac{\infty}{\infty}\right] \text{ FI}$$

$$\lim_{x \to +\infty} \frac{4}{3x^2} = \frac{4}{+\infty} = 0$$ $y=0$ as. or. dx

$$\lim_{x \to -\infty} \frac{4x-4}{x^3} = \left[\frac{\infty}{\infty}\right] FI$$

$$\lim_{x \to -\infty} \frac{4}{3x^2} = 0 \quad y=0 \text{ as. or. sx}$$

Concl. $y=0$ è as. per la f.

f) Max e min. relativi

$$y = \frac{4x-4}{x^3}$$

$$y' = \frac{4(x^3) - (4x-4) \cdot 3x^2}{x^6}$$

$$y' = \frac{4x^3 - 12x^3 + 12x^2}{x^6}$$

$$y' = \frac{-8x^3 + 12x^2}{x^6} = \frac{\cancel{x^2}(-8x+12)}{x^{\cancel{6}\,4}}$$

$$y' = \frac{-8x+12}{x^4}$$

C.N $y' = 0$

$$\frac{-8x+12}{x^4} = 0 \qquad -8x+12=0$$

$$x = \frac{12}{8} = \frac{3}{2}$$

punto crítico

C.S $f'(x) > 0$

$$\frac{-8x+12}{x^4} > 0$$

$N > 0$ $\quad -8x+12 > 0$

$$-8x > -12$$

$$8x < 12$$

$$x < \frac{3}{2}$$

D: $x^4 > 0$

$\forall x \in \mathbb{R} - \{0\}$

		0		$\frac{3}{2}$	
N	+	+	+ +	0	− −
D	+	0	+	+	+ +
y'	+	$\nexists$	+	0	−
	↗		↗	MAX	↘

$x = \frac{3}{2}$ $\quad y = \frac{6-4}{27} = 2 \cdot \frac{8}{27} = \frac{16}{27}$

$$\left(\frac{3}{2} ; \frac{16}{27}\right) \text{ max rel}$$

g) flessi $\quad f'(x) = \dfrac{-8x+12}{x^4}$

$$f''(x) = \frac{-8x^4 - (-8x+12)4x^3}{x^8}$$

$$f''(x) = \frac{-8x^4 + 32x^4 - 48x^3}{x^8}$$

$$f''(x) = \frac{24x^4 - 48x^3}{x^8}$$

$$f''(x) = \frac{\cancel{x^3}(24x-48)}{x^{\cancel{8}\,5}} = \frac{24x-48}{x^5}$$

C.N $\quad f''(x) = 0$

$$\frac{24x-48}{x^5} = 0 \qquad 24x - 48 = 0$$

$$x = 2$$

probabile punto di flesso

C.S. $f''(x) > 0$

$$\frac{24x-48}{x^5} > 0$$

N: $24x - 48 > 0$

$x > 2$

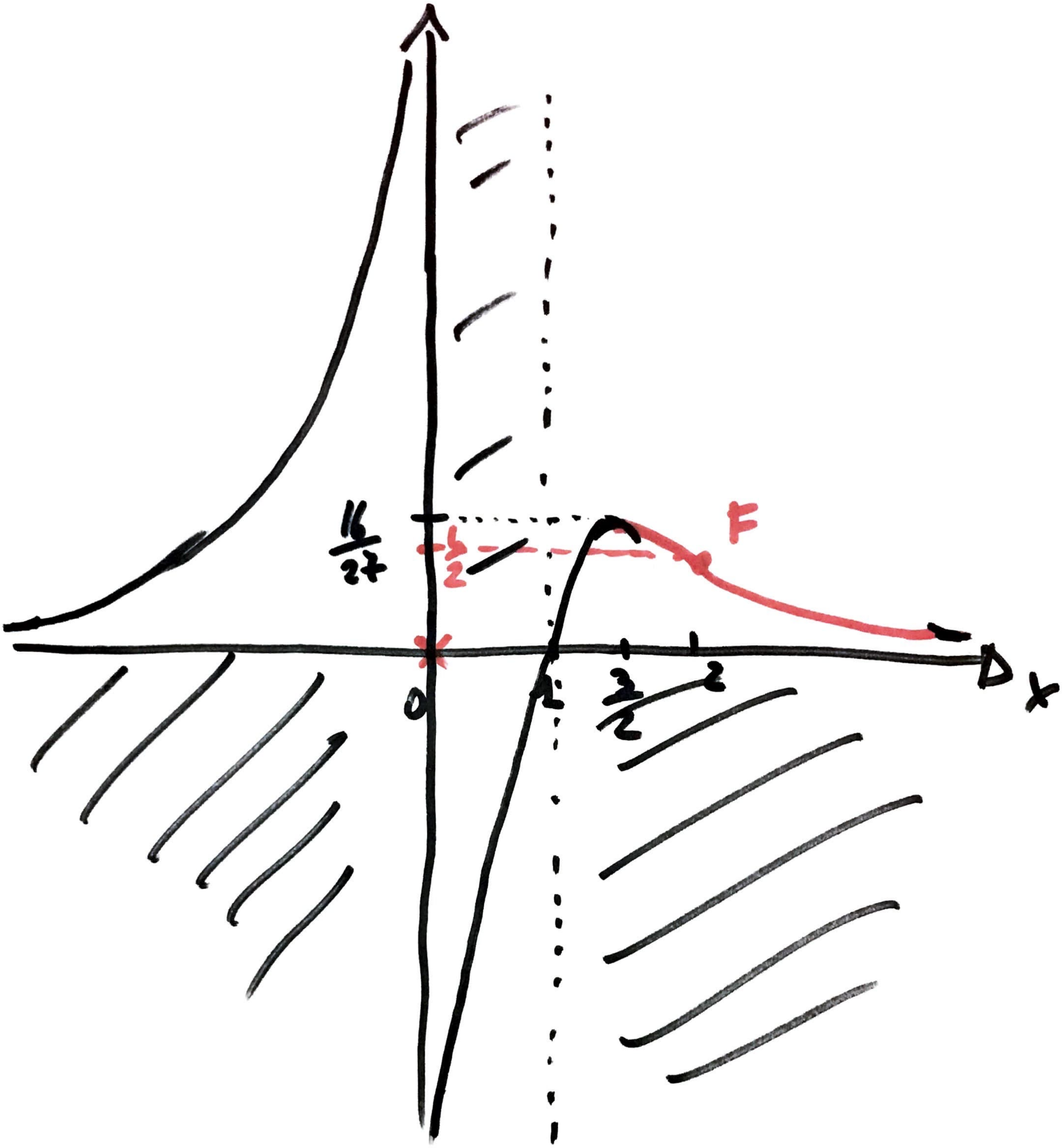
D: $x^5 > 0$

$x > 0$

		0		2	
N	$-$ $-$		$-$	0	$+$ $+$
D	$-$ $-$	0	$+$	$+$	$+$ $+$
	$+$	$\nexists$	$-$	0	$+$
	$\cup$		$\cap$	F	$\cup$

$x = 2$ $\quad y = \frac{4}{8} = \frac{1}{2}$

$\left(2 ; \frac{1}{2}\right)$ ~~F[illegible]~~ flesso



$\frac{16}{27}$
$\frac{1}{2}$
F
0
1
$\frac{3}{2}$
2
x