1) Tracciare il grafico della funzione

$$f(x) = (3x-1)e^{1-2x}$$

a) Dominio

$A = \mathbb{R}$

b) Pari o dispari

$$f(x) = (3x-1)e^{1-2x}$$

$$f(-x) = (-3x-1)e^{1+2x}$$

la f non è pari perché $f(x) \neq f(-x)$

$$-f(x) = (-3x+1)e^{1-2x}$$

la f non è dispari perché $f(-x) \neq -f(x)$

c) Intersezione assi

$$\begin{cases} y = (3x-1)\,e^{1-2x} \\ x = 0 \end{cases}$$

$$\begin{cases} y = -1e = -e \\ x = 0 \end{cases} \qquad P(0; -e)$$

$$\begin{cases} y = (3x-1)\,e^{1-2x} \\ y = 0 \end{cases}$$

$$\begin{cases} (3x-1)\,e^{1-2x} = 0 \\ y = 0 \end{cases}$$

$3x-1=0$

$x = \frac{1}{3}$

$e^{1-2x} = 0 \quad \nexists\, x \in \mathbb{R}$

$$Q\left(\frac{1}{3}; 0\right)$$

d) Segno $(f(x) > 0)$

$$(3x-1)e^{1-2x} > 0$$

I F $3x - 1 > 0 \quad x > \frac{1}{3}$

II F $e^{1-2x} > 0 \quad \forall x \in \mathbb{R}$

		$\frac{1}{3}$	
I F	$- - - -$	0	$+ + +$
II F	$+ + + +$	$+$	$+ + + +$
	$-$	0	$+$

e) Asintoti

$A = \mathbb{R}$ ($\nexists$ as. verticale)

$$\lim_{x \to +\infty} (3x-1)e^{1-2x} = [\infty \cdot 0] \ FI$$

$$a^{-n} = \frac{1}{a^n}$$

$$\lim_{x \to +\infty} (3x-1) \cdot \frac{1}{e^{-1+2x}} =$$

$$\lim_{x \to +\infty} \frac{3x-1}{e^{2x-1}} = \left[\frac{\infty}{\infty}\right] \text{FI}$$

$$\lim_{x \to +\infty} \frac{3}{e^{2x-1} \cdot (2)} = \frac{3}{+\infty} = 0$$

$y=0$ as. oriz. per la f.

$$\lim_{x \to -\infty} (3x-1) e^{1-2x} = -\infty \, (+\infty) = -\infty$$

$\nexists$ asintoto oriz. sinistro

(as. obliquo?)

f) $\overset{\text{MAX}}{y}=\overset{\text{e MIN}}{(3x-1)}\,e^{1-2x}$

$y'=\underbrace{3e^{1-2x}}+\underbrace{(3x-1)e^{1-2x}(-2)}$

$y'=e^{1-2x}\left[3-2(3x-1)\right]$

$y'=e^{1-2x}(3-6x+2)$

$y'=e^{1-2x}(5-6x) \quad A'=\mathbb{R}$

[la f è continua e derivabile in $\mathbb{R}$]

CN $\quad y'=0$

$e^{1-2x}(5-6x)=0$

$e^{1-2x}=0 \quad \nexists\, x\in\mathbb{R}$

$5-6x=0$

$-6x=-5$

C.S. $e^{1-2x}(5-6x)>0$

$e^{1-2x}>0 \quad \forall x\in\mathbb{R}$

$5-6x>0 \qquad -6x>-5$

$6x<5$

$x<\frac{5}{6}$

		$\frac{5}{6}$	
I	$+\ +\ +$	$+$	$+\ +\ +$
II	$+\ +\ +$	0	$-\ -\ -\ -\ -$
y'	$+$	0	$-$
	$\nearrow$		$\searrow$

$x=\frac{5}{6}$ max rel

$y=\left(3\cdot\frac{5}{6}-1\right)e^{1-2\cdot\frac{5}{6}}=$

$=\left(\frac{5}{2}-1\right)e^{1-\frac{5}{3}}=\frac{3}{2}e^{-\frac{2}{3}}$

$\left(\frac{5}{6};\ \frac{3}{2}e^{-\frac{2}{3}}\right)$ max rel.

g) Flessi

$$y' = e^{1-2x}(5-6x)$$

$$y'' = e^{1-2x}(-2)(5-6x) + e^{1-2x}\cdot(-6)$$

$$y'' = e^{1-2x}\left[-2(5-6x) - 6\right]$$

$$y'' = e^{1-2x}\left[-10 + 12x - 6\right]$$

$$y'' = e^{1-2x}(12x-16)$$

C.N

$$e^{1-2x}(12x-16) = 0$$

$$e^{1-2x} = 0 \quad \nexists x$$

$$12x - 16 = 0$$

$$x = \frac{16}{12} = \frac{4}{3}$$

candidato ad essere punto di flesso

C.S $e^{1-2x}(12x-16)>0$

$e^{1-2x}>0 \quad \forall x \in \mathbb{R}$

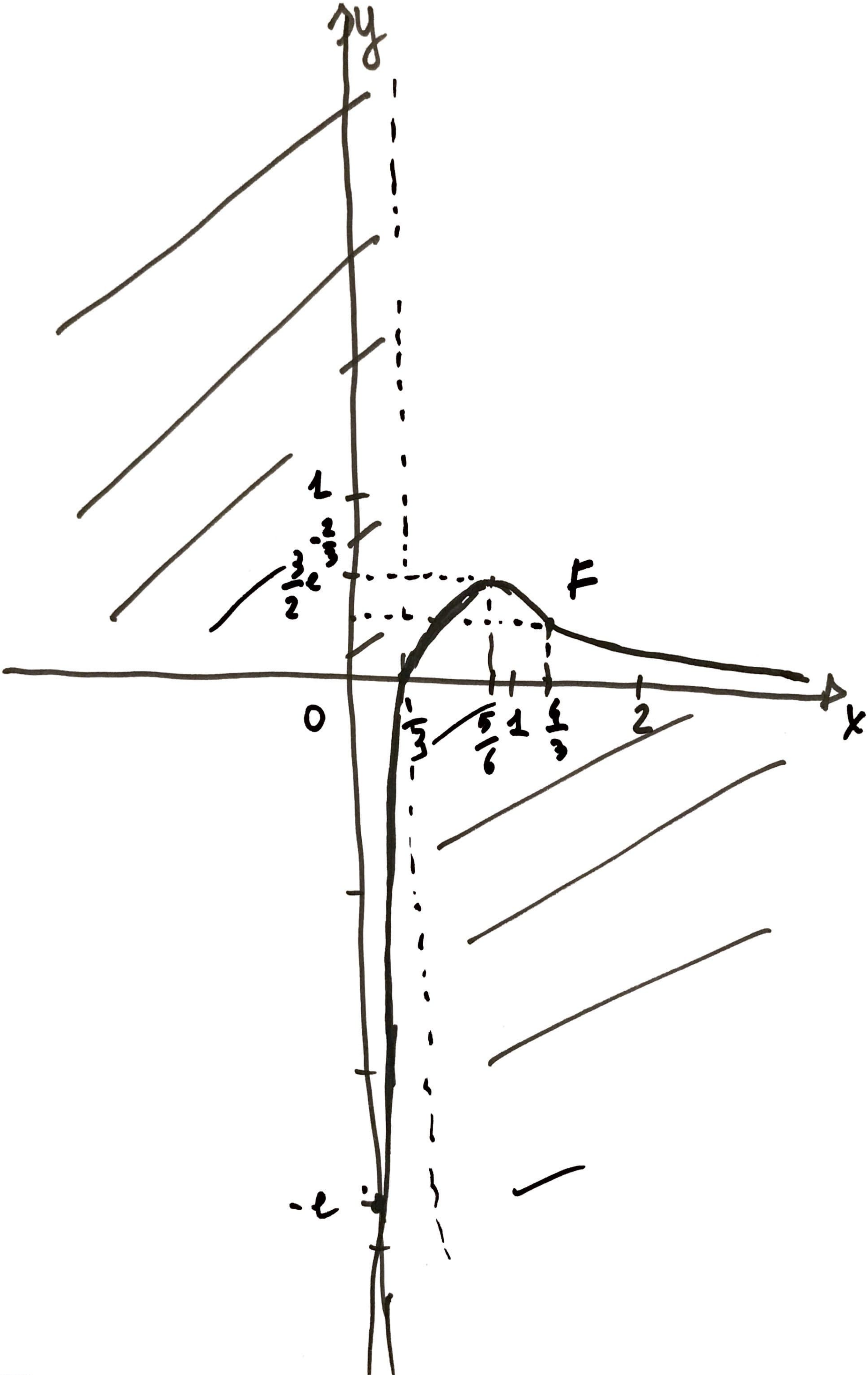
$12x-16>0 \qquad x>\frac{4}{3}$

		$\frac{4}{3}$	
I F	+ +	+	+ +
II F	− − −	0	+ +
y''	−	0	+
	$\cap$		$\cup$

$x=\frac{4}{3}$ punto di flesso

$y=3e^{1-\frac{8}{3}}=3e^{-\frac{5}{3}}$

$\left(\frac{4}{3}\,;\,3e^{-\frac{5}{3}}\right)$ flesso



y
x
0
1
F
2

2) Det. per quale valore reale positivo di K si verifica la seguente uguaglianza

$$\lim_{x \to -\infty} \frac{2x-3}{\sqrt{Kx^2+3}} = \lim_{x \to -5} \frac{x^2+3x-10}{x^2-25}$$

$$\lim_{x \to -5} \frac{x^2+3x-10}{x^2-25} = \frac{25-15-10}{25-25} = \left[\frac{0}{0}\right]$$

FI

$$\lim_{x \to -5} \frac{\cancel{(x+5)}(x-2)}{(x-5)\cancel{(x+5)}} = \frac{-7}{-10} = \frac{7}{10}$$

$$\lim_{x \to -\infty} \frac{2x-3}{\sqrt{Kx^2+3}} = \left[\frac{\infty}{\infty}\right] \text{ FI}$$

$$\lim_{x \to -\infty} \frac{x\left(2-\underbrace{\frac{3}{x}}_{\to 0}\right)}{\sqrt{x^2\left(K+\underbrace{\frac{3}{x^2}}_{\to 0}\right)}} =$$

$$\lim_{x \to -\infty} \frac{2x}{\sqrt{Kx^2}} =$$

$$\lim_{x \to -\infty} \frac{2x}{|x|\sqrt{K}} =$$

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$= \lim_{x \to -\infty} \frac{2\cancel{x}}{-\cancel{x}\sqrt{K}} = -\frac{2}{\sqrt{K}}$$

Uguagliando i due limiti si ottiene

$$-\frac{2}{\sqrt{K}} = \frac{7}{10}$$

$$-20 = 7\sqrt{K}$$

$$\sqrt{K} = -\frac{20}{7} \quad \nexists K \in \mathbb{R}^+$$

3) Determinare per via grafica il dominio della funzione

$$y = \ln\left(e^{|x|+1} - \left|\sqrt{|x|} - 2\right|\right)$$

$$\begin{cases} e^{|x|+1} - \left|\sqrt{|x|} - 2\right| > 0 \\ |x| \geq 0 \quad \text{compreso nella I dis} \end{cases}$$

$$e^{|x|+1} > \left|\sqrt{|x|} - 2\right|$$

$$f(x) = e^{|x|+1} \qquad g(x) = \left|\sqrt{|x|} - 2\right|$$

Occorre verificare graf. per quelli valori di x si ha

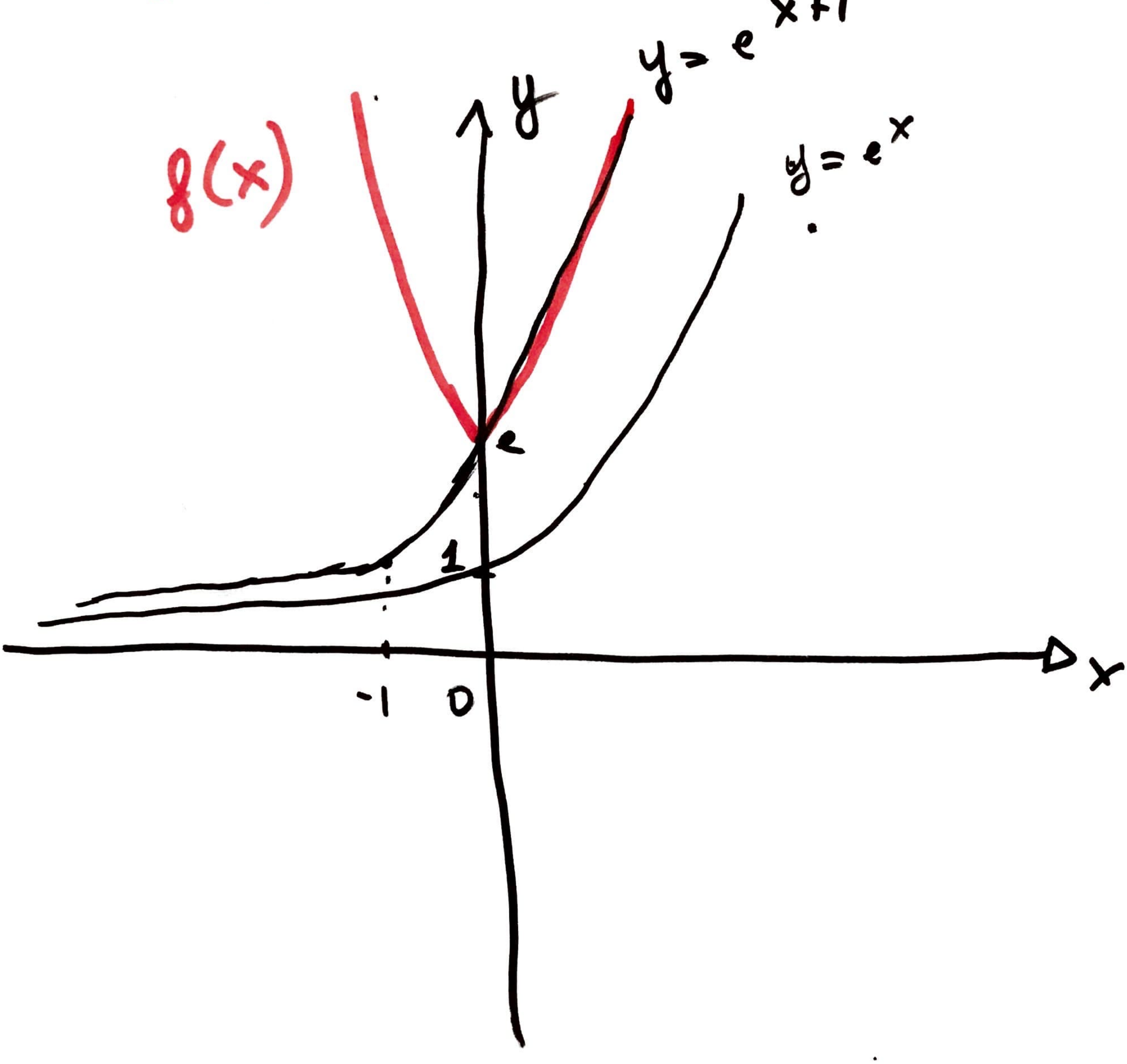
$$f > g$$

$f(x) = e^{|x|+1}$

$y = e^{x} \to y = e^{x+1} \longrightarrow y = e^{|x|+1}$

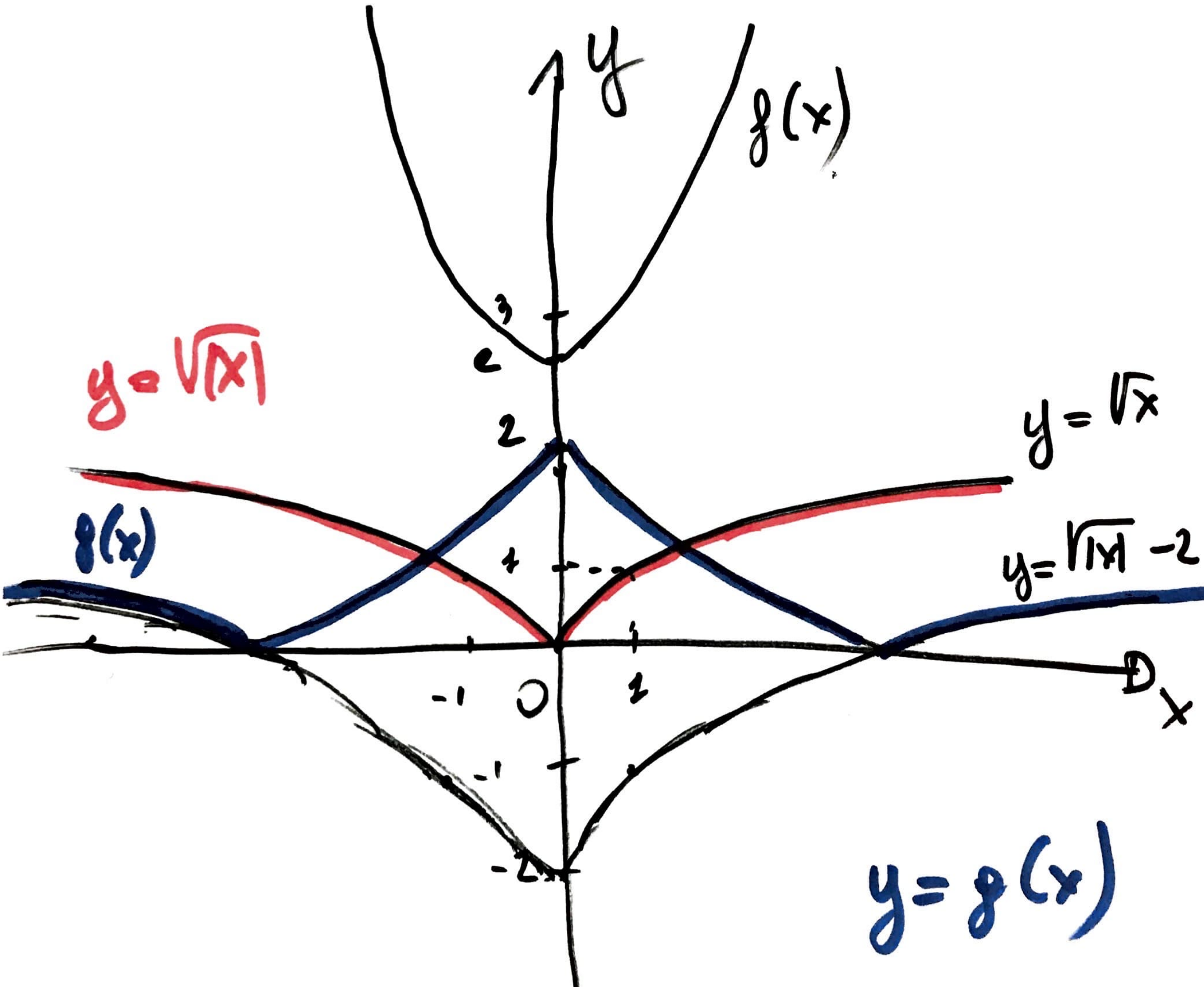
$f(x-k)$ $\quad$ $f(|x|)$

$k = -1$



$g(x) = \left| \sqrt{|x|} - 2 \right|$

$y = \sqrt{x} \underset{f(|x|)}{\longrightarrow} y = \sqrt{|x|} \underset{f(x)+K}{\longrightarrow} g = \sqrt{|x|} - 2 \underset{|f(x)|}{\longrightarrow} g = \left| \right|$



$f > g \quad \forall x \in \mathbb{R}$

$D = \mathbb{R}$

4) Dominio per via grafica

$$y=\sqrt{\frac{1}{|x|}-\ln(x-1)}$$

$$\begin{cases} \frac{1}{|x|}-\ln(x-1) \geq 0 \\ |x| \neq 0 \quad \text{comprene I} \\ x-1>0 \quad \text{comprene I} \end{cases}$$

$$\frac{1}{|x|} \geq \ln(x-1)$$

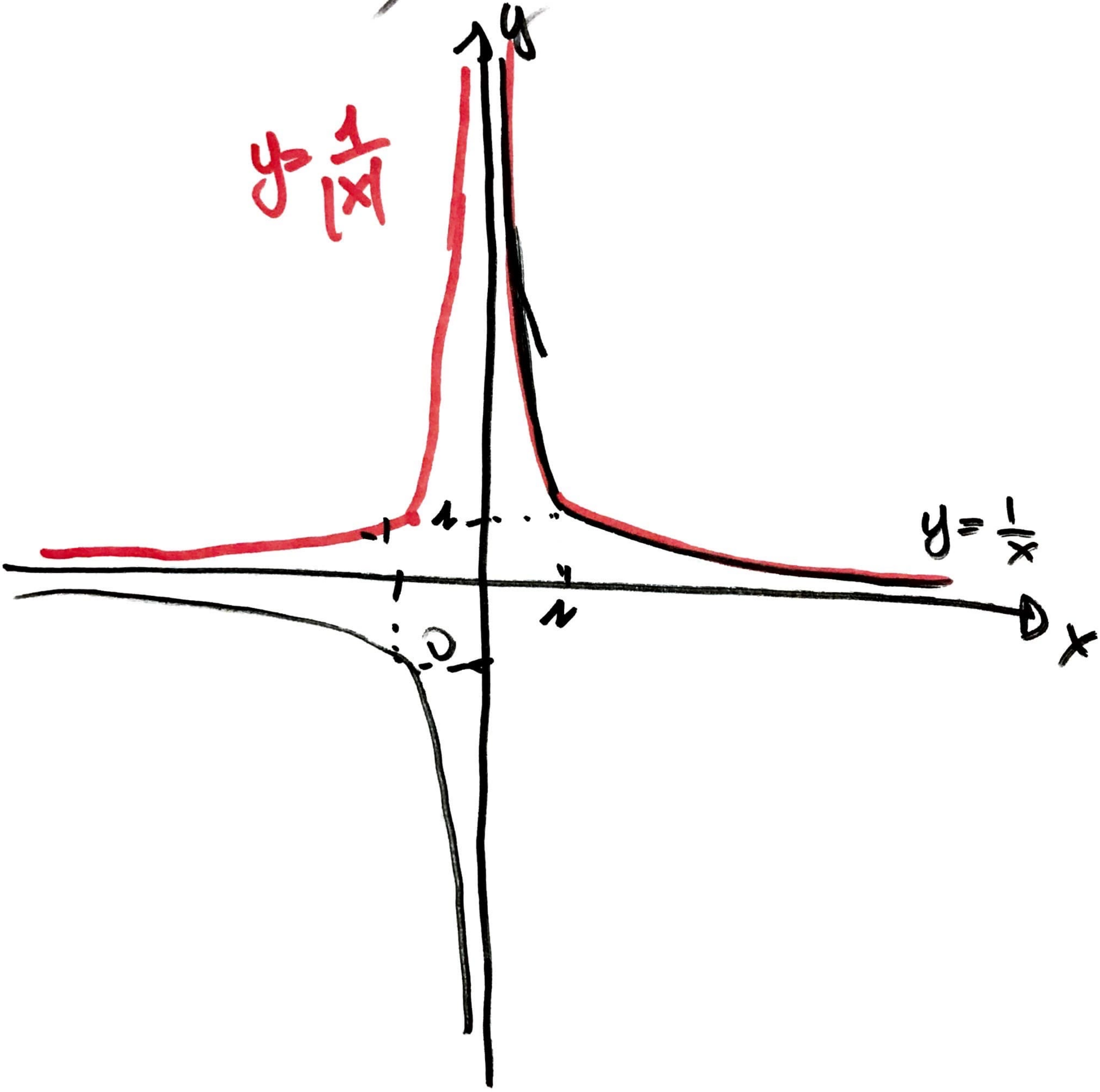
$$f(x)=\frac{1}{|x|} \qquad g(x)=\ln(x-1)$$

Si tratta di vedere per quali valori che

$$f(x) \geq g(x)$$

$$f(x) = \frac{1}{|x|}$$

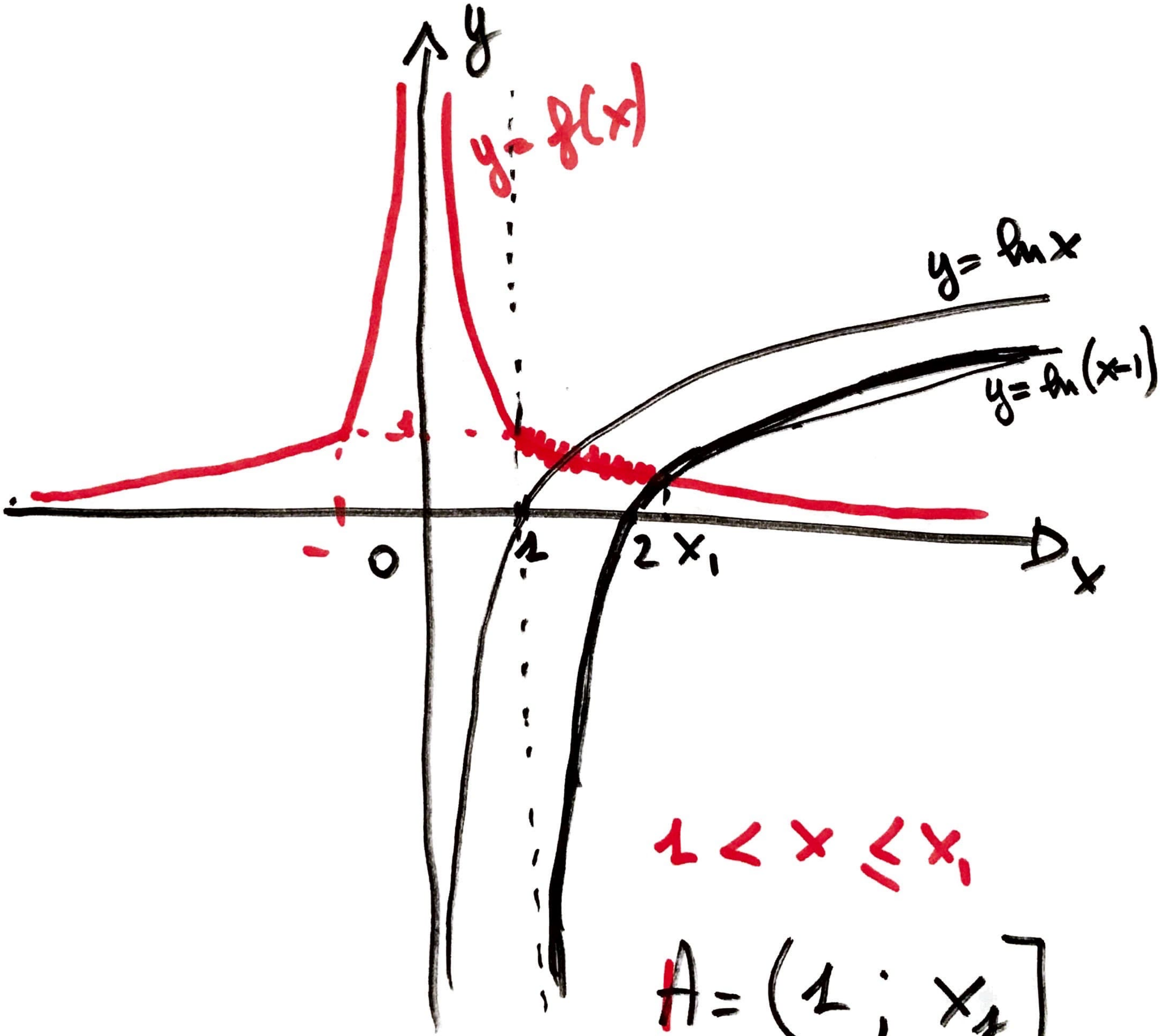
$$y = \frac{1}{x} \xrightarrow[f(|x|)]{} y = \frac{1}{|x|}$$



$f(x) = \ln(x-1)$

$y = \ln x \xrightarrow[f(x-k)]{} y = \ln(x-1)$

$k = 1$



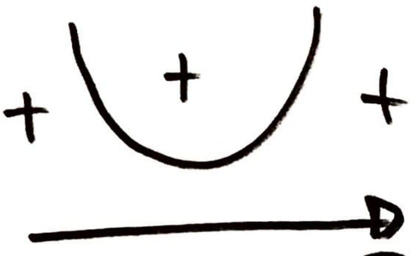
$1 < x \leq x_1$

$A = (1 ; x_1]$

5) $y = \ln(x^2-2x+2)$

a) Dominio

$x^2-2x+2 > 0$



$\forall x \in \mathbb{R}$

$x^2-2x+2=0$

$$x_{1/2} = \frac{2 \pm \sqrt{4-8}}{2}$$

$A = \mathbb{R}$

b) Pari o dispari

$f(x) = \ln(x^2-2x+2)$ $\neq$ non è pari

$f(-x) = \ln(x^2+2x+2)$

$-f(x) = -\ln(x^2+2x+2)$ $\neq$ non è dispari

c) Int. assi

$$\begin{cases} y = \ln(x^2-2x+2) \\ x=0 \end{cases} \qquad \begin{cases} y = \ln 2 \\ x=0 \end{cases} \qquad P(0, \ln 2)$$

$$\begin{cases} y = \ln(x^2-2x+2) \\ y=0 \end{cases}$$

$$\begin{cases} \ln(x^2-2x+2) = 0 \\ y=0 \end{cases}$$

$$\begin{cases} e^{\ln(x^2-2x+2)} = e^{0} \\ y=0 \end{cases}$$

$$\begin{cases} x^2-2x+2 = 1 \rightarrow x^2-2x+1=0 \\ y=0 \end{cases}$$

$$\begin{cases} (x-1)^2=0 \quad x=1 \\ y=0 \end{cases}$$

$Q(1,0)$

d) Segno

$$\ln(x^2-2x+2) > 0$$

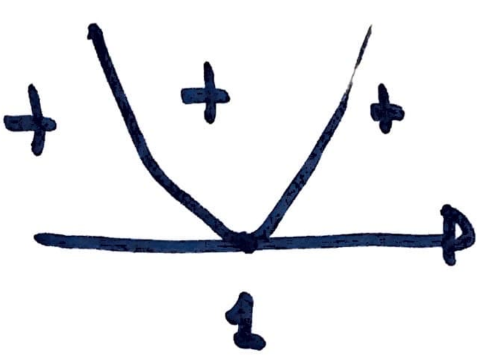
$$e^{\ln(x^2-2x+2)} > e^0$$

$$x^2-2x+2 > 1$$

$$x^2-2x+1 > 0$$

$$x^2-2x+1 = 0$$

$$x_{1/2} = \frac{2 \pm \sqrt{4-4}}{2} = 1$$



$\forall x \in \mathbb{R} - \{1\}$

e) Asintoti $(A = \mathbb{R})$

$$\lim_{x \to +\infty} \ln(x^2-2x+2) = +\infty$$

$\nexists$ as. orizz. destro

(as. obliquo ?)

$$\lim_{x\to -\infty} \ln(x^2 - 2x + 2) = +\infty$$

$\nexists$ as. oriz. sx

(as obliquo ?)

f) Max e min relativi

$$y = \ln(x^2 - 2x + 2)$$

$$y' = \frac{1}{x^2 - 2x + 2} \cdot (2x - 2)$$

$$y' = \frac{2x - 2}{x^2 - 2x + 2}$$

CN $\quad y' = 0$

$$\frac{2x-2}{x^2-2x+2} = 0 \qquad 2x - 2 = 0$$

$x = 1$ punto critico

C.S $y' > 0$

$$\frac{2x-2}{x^2-2x+2} > 0$$

N > 0

$2x-2 > 0$

$x > 1$

D $x^2-2x+2 > 0$

$x^2-2x+2 = 0$

$\forall x \in \mathbb{R}$

		1	
N	$- \; - \; -$	0	$+ \; +$
D	$+ \; + \; +$		$+ \; + \; +$
y'	$-$	0	$+$
	$\searrow$		$\nearrow$

$x = 1$ min relativo

$Q(1,0)$ min rel.

g) Flem

$$y' = \frac{2x-2}{x^2-2x+2}$$

$$y'' = \frac{2\left(x^2-2x+2\right) - \left(2x-2\right)\left(2x-2\right)}{\left(x^2-2x+2\right)^2}$$

$$y'' = \frac{2x^2-4x+4 - \left(4x^2-4x-4x+4\right)}{\left(x^2-2x+2\right)^2}$$

$$y'' = \frac{2x^2 \cancel{-4x} \cancel{+4} - 4x^2 \cancel{+4x} + 4x \cancel{-4}}{\left(x^2-2x+2\right)^2}$$

$$y'' = \frac{-2x^2+4x}{\left(x^2-2x+2\right)^2}$$

C.N $\quad y'' = 0$

$$\frac{-2x^2+4x}{(x^2-2x+2)^2} = 0 \qquad -2x^2+4x=0$$

$$2x(-x+2) = 0$$

$$2x = 0 \qquad -x+2=0$$

$$\downarrow \qquad\qquad \downarrow$$

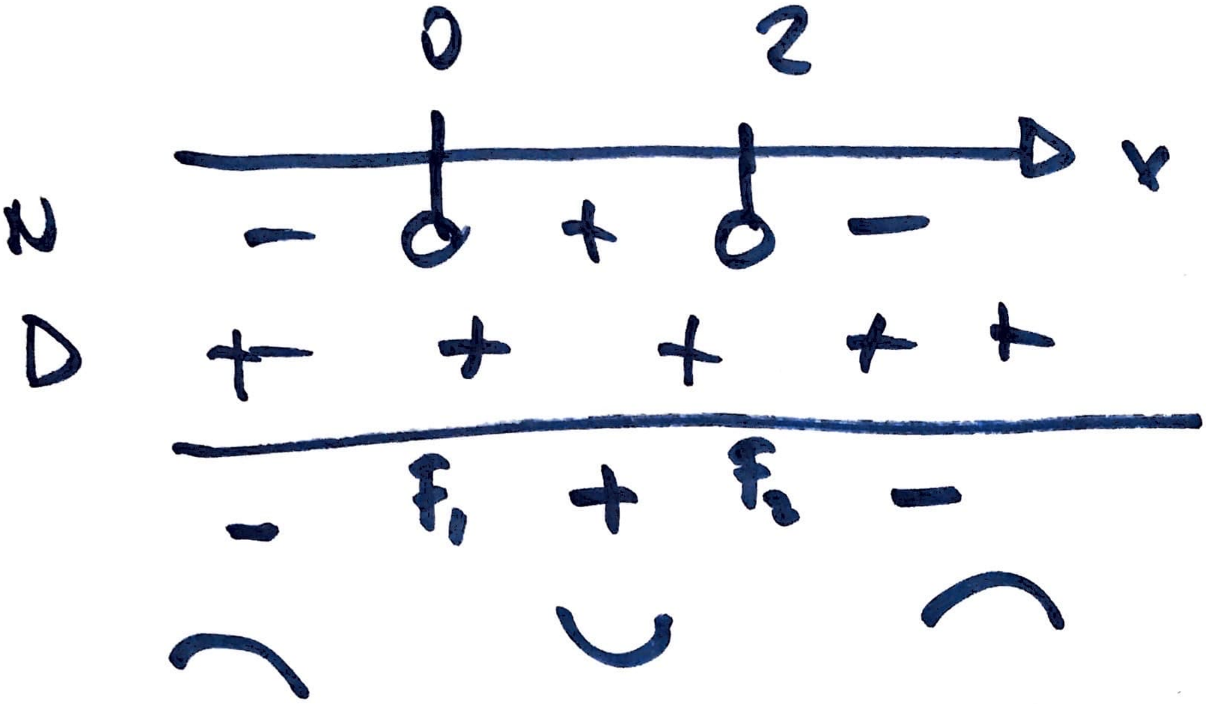
$$\underbrace{x=0 \qquad x=2}_{\text{cond. a flesso}}$$

C.S.

$$\frac{-2x^2+4x}{(x^2-2x+2)^2} > 0 \qquad N \quad -2x^2+4x>0$$

$$0 < x < 2$$

$$D: \ (x^2-2x+2)^2 > 0 \quad \forall x \in \mathbb{R}$$

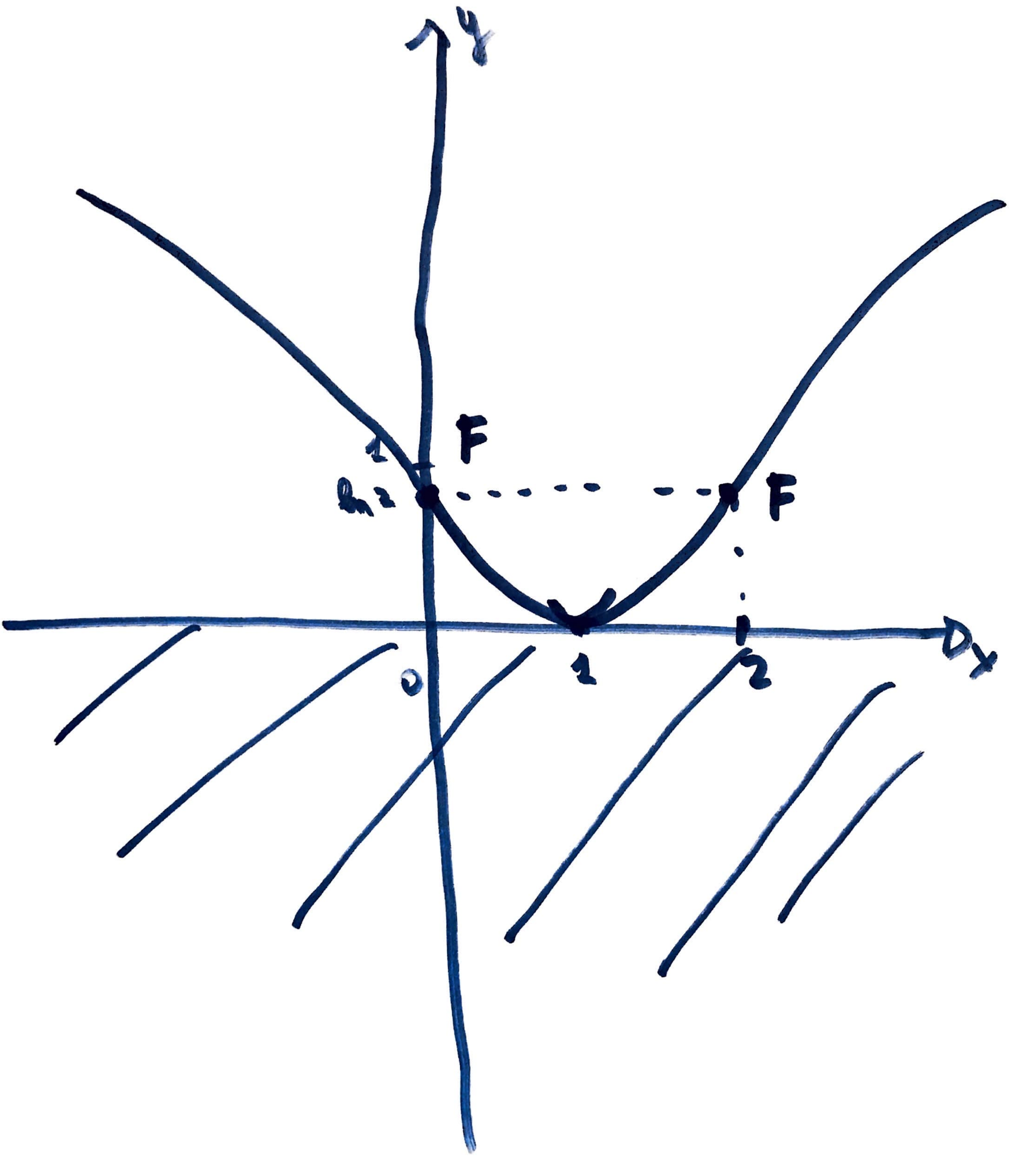


$(0\,;\,\ln 2)$ flesso

$x = 2$

$y = \ln(4 - 4 + 2) = \ln 2$

$(2, \ln 2)$ flesso


y
1
F
ln 2
F
0
1
2
x