Sia $f: \mathbb{R}^2 \to \mathbb{R}^3$ definita da $f: \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \to \begin{bmatrix} 2x_1 \\ x_1 - x_2 \\ -x_2 \end{bmatrix}$ è un'applicazione lineare? In caso positivo determ. la MATRICE TRASFORMAZIONE.

Un'applicazione è LINEARE se $f(\alpha \underline{x} + \beta \underline{y}) = \alpha f(\underline{x}) + \beta f(\underline{y})$

Sia $\underline{x} = (x_1, x_2)$ $\quad \underline{y} = (y_1, y_2)$ $\quad \alpha, \beta \in \mathbb{R}$

$$f(\alpha \underline{x} + \beta \underline{y}) = f\left(\alpha \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \beta \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}\right) = f\left(\begin{bmatrix} \alpha x_1 \\ \alpha x_2 \end{bmatrix} + \begin{bmatrix} \beta y_1 \\ \beta y_2 \end{bmatrix}\right) =$$

$$= f\left(\begin{bmatrix} \alpha x_1 + \beta y_1 \\ \alpha x_2 + \beta y_2 \end{bmatrix}\right) = \begin{bmatrix} 2(\alpha x_1 + \beta y_1) \\ \alpha x_1 + \beta y_1 - \alpha x_2 - \beta y_2 \\ -\alpha x_2 - \beta y_2 \end{bmatrix} =$$

$$= \begin{bmatrix} 2\alpha x_1 + 2\beta y_1 \\ \alpha x_1 - \alpha x_2 + \beta y_1 - \beta y_2 \\ -\alpha x_2 - \beta y_2 \end{bmatrix} = \begin{bmatrix} 2\alpha x_1 \\ \alpha x_1 - \alpha x_2 \\ -\alpha x_2 \end{bmatrix} + \begin{bmatrix} 2\beta y_1 \\ \beta y_1 - \beta y_2 \\ -\beta y_2 \end{bmatrix} =$$

$$= \alpha \begin{bmatrix} 2x_1 \\ x_1 - x_2 \\ -x_2 \end{bmatrix} + \beta \begin{bmatrix} 2y_1 \\ y_1 - y_2 \\ -y_2 \end{bmatrix} = \alpha f(\underline{x}) + \beta f(\underline{y}) \Rightarrow$$ È LINEARE.

La MATRICE CHE RAPPRESENTA f è $A = [f(e_1), f(e_2)]$

$e_1 = (1, 0) \quad e_2 = (0, 1)$

$f(e_1) = [2, 1, 0]$; $f(e_2) = [0, -1, -1]$

MATRICE $A\ (3 \times 2)$ $\quad A = \begin{bmatrix} 2 & 0 \\ 1 & -1 \\ 0 & -1 \end{bmatrix}$; infatti $A\underline{x} = f(\underline{x})$

$$A\underline{x} = A \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 1 & -1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2x_1 \\ x_1 - x_2 \\ -x_2 \end{pmatrix} = f(\underline{x})\ !$$