

CALCOLARE IL DIFFERENZIALE PRIMO DELLA FUNZIONE

$$z = \sqrt{x+y^2} \text{ NEL PUNTO } A(1,2)$$

$$\text{DIFFERENZIALE PRIMO } df(x_0, y_0) = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$df(1,2) = \frac{\partial f}{\partial x}(1,2)(x-1) + \frac{\partial f}{\partial y}(1,2)(y-2)$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x+y^2}} \rightarrow \frac{\partial f}{\partial x}(1,2) = \frac{1}{2\sqrt{1+4}} = \frac{1}{2\sqrt{5}}$$

$$\frac{\partial f}{\partial y} = \frac{2y}{2\sqrt{x+y^2}} \rightarrow \frac{\partial f}{\partial y}(1,2) = \frac{2}{\sqrt{1+4}} = \frac{2}{\sqrt{5}}$$

$$df(1,2) = \frac{1}{2\sqrt{5}}(x-1) + \frac{2}{\sqrt{5}}(y-2) = \frac{1}{2\sqrt{5}}x - \frac{1}{2\sqrt{5}} + \frac{2}{\sqrt{5}}y - \frac{4}{\sqrt{5}}$$

$$= \frac{1}{2\sqrt{5}}x + \frac{2}{\sqrt{5}}y - \frac{9}{2\sqrt{5}} = \frac{1}{\sqrt{5}}\left(\frac{1}{2}x + 2y - \frac{9}{2}\right)$$

CALCOLARE IL DIFFERENZIALE SECONDO di $z = x^6y + xy^6$ nel punto $A(1,2)$

$$d^2f(x_0, y_0) = f_{xx}(x_0, y_0)(x - x_0)^2 + 2f_{xy}(x_0, y_0)(x - x_0)(y - y_0) + f_{yy}(x_0, y_0)(y - y_0)^2$$

$$f_x = 6x^5y + y^6; \quad f_{xx} = 30x^4y \quad f_{xx}(1,2) = 60$$

$$f_{xy} = 6x^5 + 6y^5 \quad f_{xy}(1,2) = 6 + 192 = 198$$

$$f_y = x^6 + 6xy^5 \quad f_{yy} = 30xy^4 \quad f_{yy}(1,2) = 480$$

$$d^2f(1,2) = 60(x-1)^2 + 396(x-1)(y-2) + 480(y-2)^2$$