Sia $f: \mathbb{R}^3 \to \mathbb{R}^2$ definita $f: \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \to \begin{bmatrix} x_1 + x_2 - x_3 \\ 2x_1 + x_3 \end{bmatrix}$

verificare che è lineare e trovare la MATRICE ASSOCIATA.

Un'applicazione è LINEARE se $f(\alpha \underline{x} + \beta \underline{y}) = \alpha f(\underline{x}) + \beta f(\underline{y})$

Sia $\underline{x} = (x_1, x_2, x_3)$ $\quad \underline{y} = (y_1, y_2, y_3)$ $\quad \alpha, \beta \in \mathbb{R}$.

$$f(\alpha \underline{x} + \beta \underline{y}) = f\left(\alpha \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \beta \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}\right) = f\left(\begin{bmatrix} \alpha x_1 \\ \alpha x_2 \\ \alpha x_3 \end{bmatrix} + \begin{bmatrix} \beta y_1 \\ \beta y_2 \\ \beta y_3 \end{bmatrix}\right) =$$

$$= f\left(\begin{bmatrix} \alpha x_1 + \beta y_1 \\ \alpha x_2 + \beta y_2 \\ \alpha x_3 + \beta y_3 \end{bmatrix}\right) = \begin{bmatrix} \alpha x_1 + \beta y_1 + \alpha x_2 + \beta y_2 - \alpha x_3 - \beta y_3 \\ 2\alpha x_1 + 2\beta y_1 + \alpha x_3 + \beta y_3 \end{bmatrix} =$$

$$= \begin{bmatrix} \alpha(x_1 + x_2 - x_3) + \beta(y_1 + y_2 - y_3) \\ \alpha(2x_1 + x_3) + \beta(2y_1 + y_3) \end{bmatrix} = \begin{bmatrix} \alpha(x_1 + x_2 - x_3) \\ \alpha(2x_1 + x_3) \end{bmatrix} + \begin{bmatrix} \beta(y_1 + y_2 - y_3) \\ \beta(2y_1 + y_3) \end{bmatrix}$$

$$= \alpha \begin{bmatrix} x_1 + x_2 - x_3 \\ 2x_1 + x_3 \end{bmatrix} + \beta \begin{bmatrix} y_1 + y_2 - y_3 \\ 2y_1 + y_3 \end{bmatrix} = \alpha f(\underline{x}) + \beta f(\underline{y}) \Rightarrow \text{È LINEARE}$$

MATRICE ASSOCIATA ad f è $A = [f(e_1), f(e_2), f(e_3)]$

$B = \{e_1, e_2, e_3\}$ BASE CANONICA $\mathbb{R}^3$.

$e_1 = (1, 0, 0)$ $\quad e_2 = (0, 1, 0)$ $\quad e_3 = (0, 0, 1)$

$f(e_1) = [1, 2]$; $\quad f(e_2) = [1, 0]$; $\quad f(e_3) = [-1, 1]$

MATRICE (2 x 3) $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$; infatti $A\underline{x} = f(\underline{x})$

$$\begin{pmatrix} 1 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 - x_3 \\ 2x_1 + x_3 \end{pmatrix} = f(\underline{x}) \; !$$