## . PLICAZIONI LINEARI

Sia $f: \mathbb{R}^3 \to \mathbb{R}^2$ definita da $f: \begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} x+y \\ y-z \end{pmatrix}$ è un'applicazione lineare? In caso positivo determ. la MATRICE TRASFORMAZIONE.

Un'applicazione è LINEARE se $f(\alpha \underline{x} + \beta \underline{y}) = \alpha f(\underline{x}) + \beta f(\underline{y})$

Sia $\underline{x} = (x_1, x_2, x_3)$ $\quad \underline{y} = (y_1, y_2, y_3)$ $\quad \alpha, \beta \in \mathbb{R}$.

$$f(\alpha \underline{x} + \beta \underline{y}) = f\left(\alpha \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \beta \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}\right) = f\left(\begin{bmatrix} \alpha x_1 \\ \alpha x_2 \\ \alpha x_3 \end{bmatrix} + \begin{bmatrix} \beta y_1 \\ \beta y_2 \\ \beta y_3 \end{bmatrix}\right) =$$

$$f\left(\begin{bmatrix} \alpha x_1 + \beta y_1 \\ \alpha x_2 + \beta y_2 \\ \alpha x_3 + \beta y_3 \end{bmatrix}\right) = \begin{bmatrix} \alpha x_1 + \beta y_1 + \alpha x_2 + \beta y_2 \\ \alpha x_2 + \beta y_2 - \alpha x_3 - \beta y_3 \end{bmatrix} =$$

$$= \begin{bmatrix} \alpha(x_1 + x_2) + \beta(y_1 + y_2) \\ \alpha(x_2 - x_3) + \beta(y_2 - y_3) \end{bmatrix} = \begin{bmatrix} \alpha(x_1 + x_2) \\ \alpha(x_2 - x_3) \end{bmatrix} + \begin{bmatrix} \beta(y_1 + y_2) \\ \beta(y_2 - y_3) \end{bmatrix} =$$

$$\alpha \begin{bmatrix} x_1 + x_2 \\ x_2 - x_3 \end{bmatrix} + \beta \begin{bmatrix} y_1 + y_2 \\ y_2 - y_3 \end{bmatrix} = \alpha f(\underline{x}) + \beta f(\underline{y}) \Rightarrow \text{È LINEARE}$$

MATRICE TRASFORMAZIONE $\quad A = [f(e_1), f(e_2), f(e_3)]$

$e_1 = (1,0,0) \quad e_2 = (0,1,0) \quad e_3 = (0,0,1)$

$f(e_1) = [1,0]$ , $f(e_2) = [1,1]$ $\quad f(e_3) = [0,-1]$

$$\underset{(2\times 3)}{A} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}; \quad \text{infatti } A\underline{x} = f(\underline{x})$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix} \quad !$$