

DATA LA FUNZIONE $z = x e^{x+y} + 5x^5 y^2 + 2$ CALCOLARE

$$\frac{\partial^3 f}{\partial x^2 \partial y} \quad \text{e} \quad \frac{\partial^3 f}{\partial y \partial x^2} \quad (\text{ossia } f_{xx y} \text{ e } f_{yxx})$$

$$f_x = e^{x+y} + x e^{x+y} + 25x^4 y^2$$

$$f_{xx} = e^{x+y} + e^{x+y} + x e^{x+y} + 100x^3 y^2$$

$$f_{xxy} = e^{x+y} + e^{x+y} + x e^{x+y} + 200x^3 y$$

$$f_{xxy} = e^{x+y} (2+x) + 200x^3 y$$

$$f_y = x e^{x+y} + 10x^5 y$$

$$f_{yx} = e^{x+y} + x e^{x+y} + 50x^4 y$$

$$f_{yxx} = e^{x+y} + e^{x+y} + x e^{x+y} + 200x^3 y = e^{x+y} (2+x) + 200x^3 y$$

CALCOLARE IL VETTORE GRADIENTE DI $f(x,y) = \frac{xy^2 - 2xy}{x^3 - 2y^2}$ nel punto $P(1,1)$

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

$$\begin{aligned} \frac{\partial f}{\partial x} &= \frac{(y^2 - 2y)(x^3 - 2y^2) - (xy^2 - 2xy)(3x^2)}{(x^3 - 2y^2)^2} = \frac{x^3 y^2 - 2y^4 - 2x^3 y + 4y^3 - 3x^3 y^2 + 6x^3 y}{(x^3 - 2y^2)^2} \\ &= \frac{-2x^3 y^2 - 2y^4 + 4x^3 y + 4y^3}{(x^3 - 2y^2)^2} \Big|_{(1,1)} = \frac{-2 - 2 + 4 + 4}{(1-2)^2} = \frac{4}{1} = 4 \end{aligned}$$

$$\frac{\partial f}{\partial y} = \frac{(2xy - 2x)(x^3 - 2y^2) - (xy^2 - 2xy)(-4y)}{(x^3 - 2y^2)^2} \Big|_{(1,1)} = \frac{0 - (1-2)(-4)}{1} = -4$$

$$\nabla f(1,1) = (4, -4)$$