

CALCOLARE I SEGUENTI INTEGRALI

$$a) \int \frac{\log x}{x^3} dx$$

$$b) \int \frac{3x+2}{x^2-5x+4} dx$$

a) METODO DI INTEGRAZIONE X PARTI

$$\int f(x) \cdot g'(x) dx = f(x) \cdot g(x) - \int f'(x) \cdot g(x) dx$$

$$f(x) = \log x \quad f'(x) = \frac{1}{x}$$

$$g'(x) = 1/x^3 \quad g(x) = \int x^{-3} dx = \frac{x^{-2}}{-2} = -\frac{1}{2x^2}$$

$$\begin{aligned} \int \frac{\log x}{x^3} &= -\frac{1}{2x^2} \log x + \int \frac{1}{2x^3} dx = -\frac{1}{2x^2} \log x + \frac{1}{2} \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} \log x - \frac{1}{4x^2} + C \\ &= \frac{-2 \log x - 1}{4x^2} + C \end{aligned}$$

$$b) \frac{3x+2}{x^2-5x+4} = \frac{3x+2}{(x-4)(x-1)} = \frac{A}{x-4} + \frac{B}{x-1} = \frac{Ax-A+Bx-4B}{(x-4)(x-1)} =$$

$$= \frac{(A+B)x - A - 4B}{(x-4)(x-1)} \Rightarrow \begin{cases} A+B=3 \\ -A-4B=2 \end{cases} \Rightarrow \begin{cases} A=3-B \\ B-3-4B=2 \end{cases} \Rightarrow \begin{cases} A=\frac{14}{3} \\ B=-\frac{5}{3} \end{cases}$$

$$\int \frac{3x+2}{x^2-5x+4} dx = \frac{14}{3} \int \frac{1}{x-4} dx - \frac{5}{3} \int \frac{1}{x-1} dx = \frac{14}{3} \ln|x-4| -$$

$$-\frac{5}{3} \ln|x-1| + C$$