

calcolare i seguenti integrali

$$1) \int \frac{\cos(\ln x) + \ln x}{x} dx$$

$$2) \int e^x \sin x dx$$

$$3) \int \frac{\sin x}{\sqrt[3]{2+3\cos x}} dx$$

$$4) \int_0^4 (5\sqrt{x} - 4x^3) dx$$

$$1) \int \frac{1}{x} \cos(\ln x) dx + \int \frac{1}{x} \ln x dx = \sin(\ln x) + \frac{1}{2} \ln^2 x + C$$

$$2) \int e^x \sin x dx = (\text{integrazione per parti})$$

$$\begin{aligned} f &= e^x & f' &= e^x \\ g' &= \sin x & g &= -\cos x \end{aligned}$$

$$\int e^x \sin x dx = -e^x \cos x + \underbrace{\int e^x \cos x dx}_L =$$

$$= -e^x \cos x + e^x \sin x - \int e^x \sin x dx$$

$\Downarrow$

$$\begin{aligned} f &= e^x & f' &= e^x \\ g' &= \cos x & g &= \sin x \end{aligned}$$

$$= e^x \sin x - \int e^x \sin x dx$$

$$2 \int e^x \sin x = e^x \sin x - e^x \cos x$$

$$\int e^x \sin x = \frac{1}{2} e^x (\sin x - \cos x) + C$$

$$3) \int \sin x (2+3\cos x)^{-\frac{1}{3}} dx = -\frac{1}{3} \int 3 \sin x (2+3\cos x)^{-\frac{1}{3}} dx = -\frac{1}{3} \frac{(2+3\cos x)^{\frac{2}{3}}}{\frac{2}{3}}$$

$$= -\frac{1}{2} \sqrt{(2+3\cos x)^2} + C$$

$$4) 5 \int_0^4 x^{\frac{1}{2}} dx - 4 \int_0^4 x^3 dx = \left[\frac{5x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{4x^4}{4} \right]_0^4 = \left[\frac{10}{3} \sqrt{x^3} - x^4 \right]_0^4 =$$

$$= \left(\frac{10}{3} \cdot 8 - 256 \right) = \frac{80}{3} - 256 = -\frac{688}{3}$$