

HANNO DATE LE MATRICI $A = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$ $B = \begin{pmatrix} -2 & 7 \\ 1 & -3 \end{pmatrix}$ $C = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$

a) CALCOLARE A^{-1} , B^{-1} , C^{-1} ;

b) CALCOLARE ABC e $C^{-1}B^{-1}A^{-1}$; verificare che $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

a) $\det A = -1 \neq 0$ $A_{11} = 5$ $A_{12} = -2$ $A_{21} = -3$ $A_{22} = 1$ $A^{-1} = -1 \cdot \begin{vmatrix} 5 & -3 \\ -2 & 1 \end{vmatrix} = \begin{vmatrix} -5 & 3 \\ 2 & -1 \end{vmatrix}$

$\det B = -1 \neq 0$ $B_{11} = -3$ $B_{12} = -1$ $B_{21} = -7$ $B_{22} = -2$ $B^{-1} = -1 \cdot \begin{vmatrix} -3 & -7 \\ -1 & -2 \end{vmatrix} = \begin{vmatrix} 3 & 7 \\ 1 & 2 \end{vmatrix}$

$\det C = 2 \neq 0$ $C_{11} = 2$ $C_{12} = -1$ $C_{21} = -4$ $C_{22} = 3$ $C^{-1} = \frac{1}{2} \begin{vmatrix} 2 & -4 \\ -1 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ -\frac{1}{2} & \frac{3}{2} \end{vmatrix}$

b) Calcoliamo il prodotto ABC :

$AB = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} -2 & 7 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix}$; $ABC = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} =$
 $= \begin{pmatrix} 1 & 0 \\ 2 & 2 \end{pmatrix}$; $\Rightarrow ABC = \begin{pmatrix} 1 & 0 \\ 2 & 2 \end{pmatrix}$

Calcoliamo $C^{-1}B^{-1}A^{-1} = \begin{pmatrix} 1 & -2 \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix} \begin{pmatrix} 3 & 7 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & -\frac{1}{2} \end{pmatrix}$

$B^{-1}A^{-1} = \begin{pmatrix} 1 & 3 \\ 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & \frac{1}{2} \end{pmatrix}$

Calcoliamo $(ABC)^{-1}$:

$\det ABC = 2 \neq 0$ $ABC_{11} = 2$ $ABC_{12} = -2$ $ABC_{21} = 0$ $ABC_{22} = 1$ $(ABC)^{-1} = \frac{1}{2} \begin{vmatrix} 2 & 0 \\ -2 & 1 \end{vmatrix}$

$\Rightarrow (ABC)^{-1} = \begin{vmatrix} 1 & 0 \\ -1 & \frac{1}{2} \end{vmatrix}$ che è uguale a $C^{-1}B^{-1}A^{-1}$;