RMINARE I MAX e MIN VINCOLATI CON IL METODO DEI MOLTIPLICATORI DI LAGRANGE

$z = x^2+y^2$ VINCOLO $x^2+4y^2=1$

Scriviamo la FUNZIONE LAGRANGIANA

$$L(x,y,\lambda)=f(x,y)-\lambda g(x,y) = x^2+y^2-\lambda(x^2+4y^2-1)$$

$$\begin{cases} L_x=0 \\ L_y=0 \\ L_\lambda=0 \end{cases} \Rightarrow \begin{cases} 2x-2\lambda x=0 \\ 2y-8\lambda y=0 \\ -x^2-4y^2+1=0 \end{cases} \quad \begin{cases} x(1-\lambda)=0 \\ y(1-4\lambda)=0 \\ x^2+4y^2-1=0 \end{cases} \quad \begin{cases} x=0 \\ y=0 \\ -1=0 \end{cases} \quad \emptyset$$

NON ESISTONO MAX O MIN VINCOLATI.

$z = x-y+3$ VINCOLO $x^2+y^2-x+y=0$

$$L(x,y,\lambda) = x-y+3-\lambda(x^2+y^2-x+y)=0$$

$$\begin{cases} L_x=0 \\ L_y=0 \\ L_\lambda=0 \end{cases} \Rightarrow \begin{cases} 1-2\lambda x+\lambda=0 \\ -1-2\lambda y-\lambda=0 \\ x^2+y^2-x+y=0 \end{cases} \quad \begin{cases} x=\dfrac{\lambda+1}{2\lambda} \\ y=\dfrac{-1-\lambda}{2\lambda} \\ / \end{cases} \quad \begin{cases} / \\ / \\ \left(\dfrac{\lambda+1}{2\lambda}\right)^2+\left(\dfrac{-1-\lambda}{2\lambda}\right)^2-\dfrac{\lambda+1}{2\lambda}+\dfrac{-1-\lambda}{2\lambda}=0 \end{cases}$$

$$\begin{cases} = \\ \dfrac{(\lambda+1)^2}{4\lambda^2}+\dfrac{(-1-\lambda)^2}{4\lambda^2}-\dfrac{\lambda+1}{2\lambda}-\dfrac{1+\lambda}{2\lambda}=0 \end{cases} \quad \begin{cases} - \\ - \\ \lambda^2+2\lambda+1+1+\lambda^2+2\lambda-2\lambda(\lambda+1)-2\lambda(1+\lambda)=0 \end{cases}$$

$$\begin{cases} / \\ / \\ \cancel{2\lambda^2}+\cancel{4\lambda}+2\cancel{-2\lambda^2}\cancel{-2\lambda}\cancel{-2\lambda}-2\lambda^2=0 \end{cases} \quad \begin{cases} = \\ -2\lambda^2+2=0 \end{cases} \quad \lambda^2-1=0 \quad \lambda=\pm 1$$

$$\begin{cases} x=0 \\ y=0 \\ \lambda=-1 \end{cases} \vee \begin{cases} x=1 \\ y=-1 \\ \lambda=+1 \end{cases} \Rightarrow \begin{matrix} P(0,0,-1). \\ Q(1,-1,1) \end{matrix}$$

Scrivo la matrice D

$$D=\begin{pmatrix} 0 & g_x & g_y \\ g_x & L_{xx} & L_{xy} \\ g_y & L_{yx} & L_{yy} \end{pmatrix} = \begin{pmatrix} 0 & 2x-1 & 2y+1 \\ 2x-1 & -2\lambda & 0 \\ 2y+1 & 0 & -2\lambda \end{pmatrix}$$

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$$D(0,0,-1)=\left(\begin{array}{ccc} 0 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{array}\right)\begin{array}{cc} 0 & -1 \\ -1 & 2 \\ 1 & 0 \end{array} = -2-2=-4<0 \Rightarrow P(0,0)$$

MIN REL. VINCOLATO

$$D(1,-1,1)=\left(\begin{array}{ccc} 0 & 1 & -1 \\ 1 & -2 & 0 \\ -1 & 0 & -2 \end{array}\right)\begin{array}{cc} 0 & 1 \\ 1 & -2 \\ -1 & 0 \end{array} = -(-2-2)=4>0 \quad Q(1,-1)$$

MAX REL. VINCOLATO