

MINIMARE I MAX e MIN VINCOLATI DELLA FUNZIONE $Z = x^2 + 2y^2$
CON IL VINCOLO $x^2 + y^2 = 1$ (CON IL METODO MOLTIPLICATORI LAGRANGE)

$$L(x, y, \lambda) = x^2 + 2y^2 - \lambda(x^2 + y^2 - 1)$$

$$\begin{cases} L_x = 0 \\ L_y = 0 \\ L_\lambda = 0 \end{cases} \Rightarrow \begin{cases} 2x - 2\lambda x = 0 \\ 4y - 2\lambda y = 0 \\ x^2 + y^2 - 1 = 0 \end{cases} \begin{cases} x - \lambda x = 0 \\ 2y - \lambda y = 0 \\ / \end{cases} \begin{cases} x(1-\lambda) = 0 & x=0 \\ y(2-\lambda) = 0 & y=0 \\ -1 = 0 \end{cases} \quad \text{NON ESISTONO MAX O MIN VINCOLATI}$$

$Z = x^2 + y^2$ VINCOLO $x^2 y - 2 = 0$

$$L(x, y, \lambda) = x^2 + y^2 - \lambda(x^2 y - 2)$$

$$\begin{cases} L_x = 0 \\ L_y = 0 \\ L_\lambda = 0 \end{cases} \begin{cases} 2x - 2\lambda xy = 0 \\ 2y - \lambda x^2 = 0 \\ x^2 y - 2 = 0 \end{cases} \begin{cases} x - \lambda xy = 0 \\ 2y - \lambda x^2 = 0 \\ / \end{cases} \begin{cases} x(1 - \lambda y) = 0 & x=0 \\ 2y - \lambda x^2 = 0 & y = \frac{1}{\lambda} \end{cases}$$

$$\begin{cases} x=0 \\ y=0 \\ 0-2=0 \end{cases} \quad \vee \quad \begin{cases} y = \frac{1}{\lambda} \\ \frac{2}{\lambda} - \lambda x^2 = 0 \end{cases} \begin{cases} y = \frac{1}{\lambda} \\ 2 - \lambda^2 x^2 = 0 & x^2 = \frac{2}{\lambda^2} \\ \frac{2}{\lambda^2} \cdot \frac{1}{\lambda} - 2 = 0 \end{cases} \begin{cases} x = \pm \sqrt{2} \\ y = 1 \\ \lambda = 1 \end{cases}$$

$P(\sqrt{2}, 1, 1)$ $Q(-\sqrt{2}, 1, 1)$ Scrivo la matrice D

$$D = \begin{pmatrix} 0 & g_x & g_y \\ g_x & L_{xx} & L_{xy} \\ g_y & L_{yx} & L_{yy} \end{pmatrix} = \begin{pmatrix} 0 & 2xy & x^2 \\ 2xy & 2 - 2\lambda y & -2\lambda x \\ x^2 & -2\lambda x & 2 \end{pmatrix}$$

$$D(\sqrt{2}, 1, 1) = \begin{pmatrix} 0 & 2\sqrt{2} & 2 \\ 2\sqrt{2} & 0 & -2\sqrt{2} \\ 2 & -2\sqrt{2} & 2 \end{pmatrix} \begin{pmatrix} 0 & 2\sqrt{2} \\ 2\sqrt{2} & 0 \\ 2 & -2\sqrt{2} \end{pmatrix} = -16 - 16 - (16) < 0$$

$P(\sqrt{2}, 1)$ MIN. VINC.

$$D(-\sqrt{2}, 1, 1) = \begin{pmatrix} 0 & -2\sqrt{2} & 2 \\ -2\sqrt{2} & 0 & 2\sqrt{2} \\ 2 & 2\sqrt{2} & 2 \end{pmatrix} \begin{pmatrix} 0 & -2\sqrt{2} \\ -2\sqrt{2} & 0 \\ 2 & 2\sqrt{2} \end{pmatrix} = -16 - 16 - 16 < 0$$

$P(-\sqrt{2}, 1)$ MIN VINC.