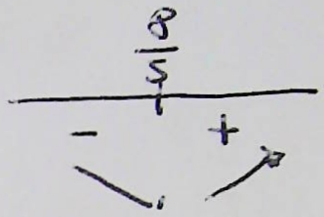
3) MAX e MIN RELATIVI di

$z = x^2+y^2$ con il vincolo $2x-y=4$

$z = x^2+y^2$ D: $\mathbb{R}^2$ esplicito il vincolo $y=2x-4$ e sostituisco $F(x) = x^2+(2x-4)^2 = x^2+4x^2-16x+16 = 5x^2-16x+16$

$F'(x) = 10x-16$ $\quad F'(x)>0 \quad 10x-16>0 \quad x>\frac{8}{5}$



$\begin{cases} x=\frac{8}{5} \\ y=\frac{16}{5}-4 \end{cases} \quad \begin{cases} x=\frac{8}{5} \\ y=-\frac{4}{5} \end{cases}$ MINIMO RELATIVO.

$z = x+y$ con il vincolo $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{2}$ (METODO LAGRANGIANA)

$L(x,y,\lambda) = f(x,y)-\lambda g(x,y) = x+y-\lambda\left(\frac{1}{x^2}+\frac{1}{y^2}-\frac{1}{2}\right)$

$$\begin{cases} L_x=0 \\ L_y=0 \\ L_\lambda=0 \end{cases} \Rightarrow \begin{cases} 1+\frac{2}{x^3}\lambda=0 \\ 1+\frac{2\lambda}{y^3}=0 \\ \frac{1}{x^2}+\frac{1}{y^2}-\frac{1}{2}=0 \end{cases} \quad \begin{cases} x=-\sqrt[3]{2\lambda} \\ y=-\sqrt[3]{2\lambda} \\ \frac{1}{(-\sqrt[3]{2\lambda})^2}+\frac{1}{(-\sqrt[3]{2\lambda})^2}-\frac{1}{2}=0 \end{cases} \quad \begin{cases} / \\ / \\ \frac{2}{\sqrt[3]{4\lambda^2}}-\frac{1}{2}=0 \end{cases}$$

$$\begin{cases} / \\ / \\ \frac{\sqrt[3]{4\lambda^2}}{2}=\frac{2}{1} \end{cases} \quad \begin{cases} \\ \\ \sqrt[3]{4\lambda^2}=4 \end{cases} \quad \begin{cases} \\ \\ 4\lambda^2=64 \end{cases} \quad \begin{cases} \\ \\ \lambda^2=16 \end{cases} \quad \lambda=\pm 4$$

$\begin{cases} x=-2 \\ y=-2 \\ \lambda=4 \end{cases} \vee \begin{cases} x=2 \\ y=2 \\ \lambda=-4 \end{cases}$ $\quad P(-2,-2,4)$ $\quad Q(2,2,-4)$ Scrivo la matrice

$$D: \begin{pmatrix} 0 & g_x & g_y \\ g_x & L_{xx} & L_{xy} \\ g_y & L_{xy} & L_{yy} \end{pmatrix} = \begin{pmatrix} 0 & -\frac{2}{x^3} & -\frac{2}{x^3} \\ -\frac{2}{x^3} & -\frac{6\lambda}{x^4} & 0 \\ -\frac{2}{x^3} & 0 & -\frac{6\lambda}{y^4} \end{pmatrix};$$

$$D(-2,-2,4) = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{3}{2} & 0 \\ \frac{1}{4} & 0 & -\frac{3}{2} \end{pmatrix} \begin{matrix} 0 & \frac{1}{4} \\ \frac{1}{4} & -\frac{3}{2} \\ \frac{1}{4} & 0 \end{matrix} = -\left(-\frac{3}{32}-\frac{3}{32}\right) = \frac{3}{16}>0$$

$P(-2,-2)$ MAX REL. VINCOLATO

$$D(2,2,-4) = \begin{pmatrix} 0 & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & 3/2 & 0 \\ -\frac{1}{4} & 0 & 3/2 \end{pmatrix} \begin{matrix} 0 & -\frac{1}{4} \\ -\frac{1}{4} & 3/2 \\ -\frac{1}{4} & 0 \end{matrix} = -\left(+\frac{3}{32}+\frac{3}{32}\right) = -\frac{3}{16}<0$$

$Q(+2,2)$ MIN REL. VINC.