

3) CONTINUITÀ

(4)

$$\lim_{x \rightarrow 1^-} \frac{ax^2 - 1}{x + b} = \frac{a-1}{1+b}$$

$$\lim_{x \rightarrow 1^+} \ln x + 1 = 1$$

$$\Rightarrow \frac{a-1}{b+1} = 1 \Rightarrow a-1 = b+1$$

$$\boxed{a = b+2}$$

DERIVABILITÀ

$$f'(x) = \begin{cases} \frac{2ax(x+b) - ax^2 + 1}{(x+b)^2} = \frac{2ax^2 + 2abx - ax^2 + 1}{(x+b)^2} = \frac{ax^2 + 2abx + 1}{(x+b)^2} \\ \frac{1}{x} \quad x > 1 \end{cases}$$

$$\lim_{x \rightarrow 1^+} \frac{1}{x} = 1 ; \lim_{x \rightarrow 1^-} \frac{ax^2 + 2abx + 1}{(x+b)^2} = \boxed{\frac{a + 2ab + 1}{(1+b)^2} = 1}$$

$$\begin{cases} a = b+2 \\ b+2 + 2b^2 + 4b + 1 = 1 + 2b + b^2 \end{cases} \Rightarrow \begin{cases} a = b+2 \\ b^2 + 3b + 2 = 0 \quad b = \frac{-3 \pm \sqrt{9-8}}{2} \end{cases}$$

$$\begin{cases} a = b+2 \\ b = -1 \end{cases} \quad \begin{cases} a = 0 \\ b = -2 \end{cases} \quad \begin{cases} a = 1 \\ b = -1 \end{cases}$$

4) $\lim_{x \rightarrow 1^-} f(x) = a+2$

$$\Rightarrow a+2 = 2a+1 \quad \boxed{a=1}$$

$$\lim_{x \rightarrow 1^+} f(x) = 2a+1$$

$$f'(x) = \begin{cases} \frac{x}{\sqrt{x^2+3}} & x \leq 1 \\ \frac{b}{x} + 2a+1 & x > 1 \end{cases}$$

$$\lim_{x \rightarrow 1^-} f'(x) = \frac{1}{2}$$

$$\boxed{b + 2a + 1 = \frac{1}{2}}$$

$$\lim_{x \rightarrow 1^+} f'(x) = b + 2a + 1$$

$$\begin{cases} a = 1 \\ b + 3 = \frac{1}{2} \end{cases} \Rightarrow \begin{cases} a = 1 \\ b = -\frac{5}{2} \end{cases}$$