Esercitazione 4 - Soluzione

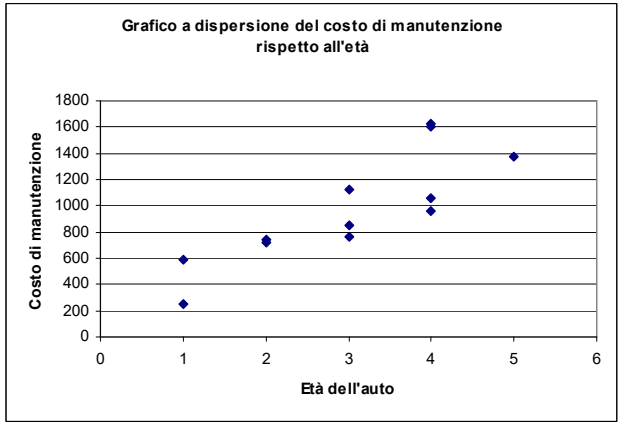
1.

Età x_i	Costo y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$
1	253	-2	-719	1438	4	516961
1	592	-2	-380	760	4	144400
2	741	-1	-231	231	1	53361
2	722	-1	-250	250	1	62500
3	851	0	-121	0	0	14641
3	767	0	-205	0	0	42025
3	1120	0	148	0	0	21904
4	1055	1	83	83	1	6889
4	959	1	-13	-13	1	169
4	1625	1	653	653	1	426409
4	1600	1	628	628	1	394384
5	1379	2	407	814	4	165649
36	**11664**	**0**	**0**	**4844**	**18**	**1849292**

$\bar{x} = 36/12 = 3$

$\bar{y} = 11664/12 = 972$

$$\rho = \frac{4844}{\sqrt{18 \cdot 1849292}} = 0{,}840$$



2.

Investimenti x_i	Fatturato y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$
17,270	2128,020	-1,084	-245,085	265,623	1,175	60066,461
18,006	2430,049	-0,348	56,944	-19,805	0,121	3242,665
13,428	1623,967	-4,926	-749,138	3690,102	24,264	561207,144
14,123	1799,352	-4,231	-573,753	2427,433	17,900	329192,046
28,942	3884,135	10,588	1511,030	15999,092	112,110	2283212,870
91,769	**11865,523**	**0**	**0**	**22362,444**	**155,569**	**3236921,185**

$\bar{x} = 91{,}769/5 = 18{,}354 \qquad \bar{y} = 11865{,}523/5 = 2373{,}105$

$$\hat{\beta}_1 = \frac{22362{,}444}{155{,}569} = 143{,}75$$

$$\hat{\beta}_0 = 2373{,}105 - 143{,}75 \cdot 18{,}354 = -265{,}188$$

$$R^2 = \frac{22362{,}444^2}{155{,}569 \cdot 3236921{,}185} = 0{,}993$$

$$\hat{y} = -265{,}188 + 143{,}75 \cdot 18 = 2322{,}247$$

3.

D_i = {pezzo difettoso alla i-esima estrazione}

a) $P(D_1 \cap D_2^c) = P(D_2^c|D_1)\, P(D_1) = 8/9 \cdot 2/10 = 8/45$
b) $P(D_1^c \cap D_2) = P(D_2|D_1^c)\, P(D_1^c) = 2/9 \cdot 8/10 = 8/45$
c) $P[(D_1^c \cap D_2) \cup (D_1 \cap D_2^c)] = P(D_1^c \cap D_2) + P(D_1 \cap D_2^c) = 16/45$
d) $P(D_1 \cap D_2) = P(D_2|D_1)\, P(D_1) = 1/9 \cdot 2/10 = 1/45$
e) $P(D_1^c \cap D_2^c) = P(D_2^c|D_1^c)\, P(D_1^c) = 7/9 \cdot 8/10 = 28/45$

4.

A = {compagnia X perde il mio bagaglio}
B = {compagnia Y perde il mio bagaglio}
P(A) > P(B)
$P(A \cap B) = P(A) \cdot P(B) = 0{,}0002$
$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0{,}03$

$$\begin{cases} P(A)\cdot P(B) = 0{,}0002 \\ P(A) + P(B) - 0{,}0002 = 0{,}03 \end{cases}$$

$$\begin{cases} P(A)\cdot P(B) = 0{,}0002 \\ P(A) = 0{,}0302 - P(B) \end{cases}$$

$$\begin{cases} [0{,}0302 - P(B)]\cdot P(B) = 0{,}0002 \\ P(A) = 0{,}0302 - P(B) \end{cases}$$

$$\begin{cases} P(B)^2 - 0{,}0302P(B) + 0{,}0002 = 0 \\ P(A) = 0{,}0302 - P(B) \end{cases}$$

$$\begin{cases} P(B) = \dfrac{0{,}0302 \pm \sqrt{0{,}0302^2 - 4\cdot 0{,}0002}}{2} \\ P(A) = 0{,}0302 - P(B) \end{cases}$$

$$\begin{cases} P(B) = \dfrac{0{,}0098}{0{,}0204} \\ P(A) = \dfrac{0{,}0204}{0{,}0098} \end{cases}$$

P(A) = 0,0204 P(B) = 0,0098

5.

B = {il cliente ha visto la campagna pubblicitaria}
A = {il cliente ha acquistato il servizio}
P(A) = 0,6
P(B|A) = 0,7
$P(B|A^c) = 0{,}2$

P(A|B)>P(A)?

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B \mid A)P(A) + P(B \mid A^c)P(A^c)} = \frac{0{,}7 \cdot 0{,}6}{0{,}7 \cdot 0{,}6 + 0{,}2 \cdot 0{,}4} = 0{,}84$$

6.

C = {le vendite di personal computer aumentano}
S = {le vendite di software aumentano}

P(C) = 0,4
P(S) = 0,5
$P(C \cap S) = 0{,}2$

a) $P(C|S) = P(C \cap S) / P(S) = 0{,}2 / 0{,}5 = 0{,}4$
b) $P(S|C) = P(C \cap S) / P(C) = 0{,}2 / 0{,}4 = 0{,}5$
c) Si, infatti osservando il risultato al punto a) concludiamo che gli eventi sono indipendenti e quindi P(S|C) = P(S) = 0,5.