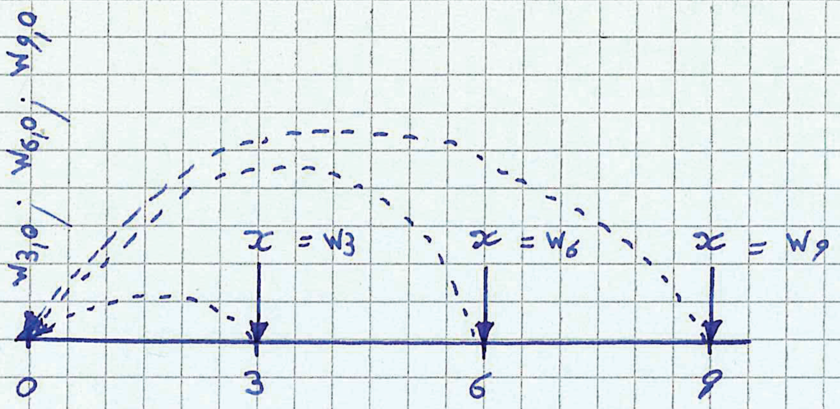
3.58



$$W_3 = W_{3,0}\left(1 + i\,\frac{3}{12}\right)$$

$$x = W_{3,0}\left(1 + \frac{1}{4} i\right); \qquad W_{3,0} = \frac{x}{1 + \frac{1}{4} i}$$

$$W_6 = W_{6,0}\left(1 + i\,\frac{6}{12}\right)$$

$$x = W_{6,0}\left(1 + i\,\frac{1}{2}\right); \qquad W_{6,0} = \frac{x}{1 + \frac{1}{2} i}$$

$$W_9 = W_{9,0}\left(1 + i\,\frac{9}{12}\right)$$

$$x = W_{9,0}\left(1 + \frac{3}{4} i\right); \qquad W_{9,0} = \frac{x}{1 + \frac{3}{4} i}$$

$$W_{3,0} + W_{6,0} + W_{9,0} = 1000\ \$$$

$$\frac{x}{1 + \frac{1}{4} i} + \frac{x}{1 + \frac{1}{2} i} + \frac{x}{1 + \frac{3}{4} i} = 1000\ \$$$

$$\left(\frac{1}{1 + \frac{1}{4} \cdot 0.11} + \frac{1}{1 + \frac{1}{2} \cdot 0.11} + \frac{1}{1 + \frac{3}{4} \cdot 0.11}\right) x = 1000\ \$$$

$$x = 351.51\ \$$$

8

$$\min f(x,y) = 2x^2 + y^2 - xy + x - y$$

$$\nabla f = (4x - y + 1,\ 2y - x - 1)^T$$

$$\nabla f = \underline{0}$$

$$\begin{cases} 4x - y + 1 = 0 \\ 2y - x - 1 = 0 \end{cases} \qquad \begin{cases} y = 4x + 1 \\ 2(4x+1) - x - 1 = 0 \end{cases}$$

$$\begin{cases} y = 4x + 1 \\ 8x + 2 - x - 1 = 0 \end{cases} \qquad \begin{cases} y = 4x + 1 \\ 7x + 1 = 0 \end{cases}$$

$$\begin{cases} y = -\frac{4}{7} + 1 = \frac{3}{7} \\ x = -\frac{1}{7} \end{cases} \qquad A\left(-\frac{1}{7}, \frac{3}{7}\right)$$

$$f''_{xx} = 4 \qquad f''_{xy} = -1$$

$$f''_{yx} = -1 \qquad f''_{yy} = 2$$

$$\underline{\underline{H}}f = \begin{bmatrix} 4 & -1 \\ -1 & 2 \end{bmatrix} = \underline{\underline{H}}f(A)$$

$$H_1 = 4 > 0$$

$$H_2 = \det \underline{\underline{H}}f(A) = 4 \cdot 2 - (-1)(-1) = 8 - 1 = 7 > 0$$

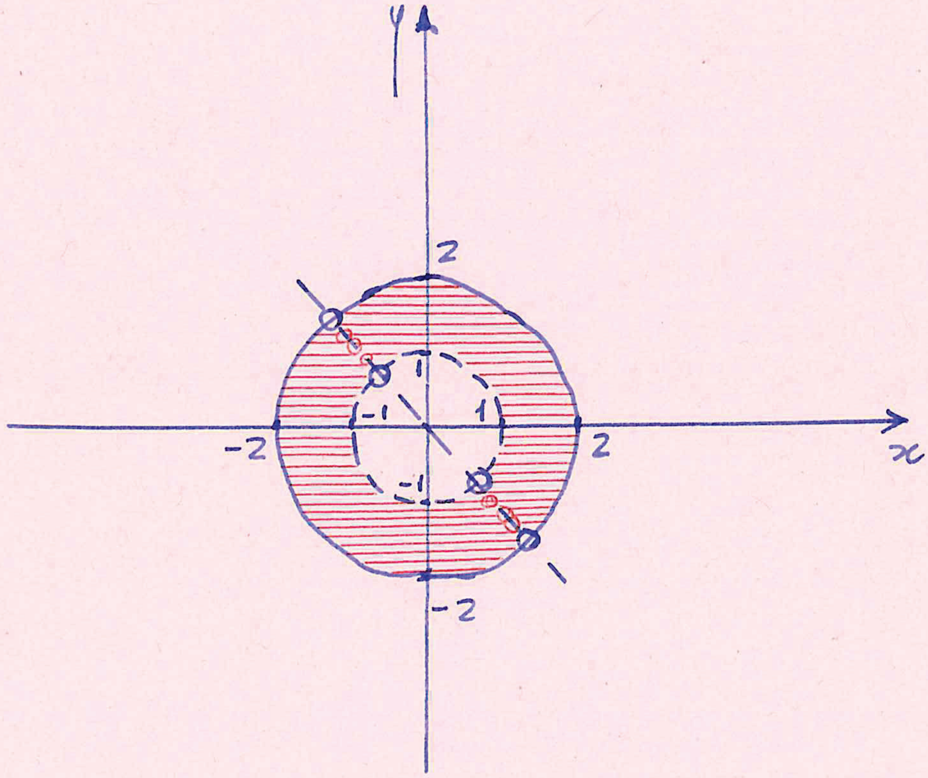
$$\underline{\underline{H}}f(A) > 0$$

A is a local minimum.

Draw the domain of the following function:

$$f(x,y) = \ln(x^2+y^2-1) - \frac{\sqrt{4-x^2-y^2}}{x+y}$$

$$\begin{cases} x^2+y^2-1 > 0 \\ 4-x^2-y^2 \geq 0 \\ x+y \neq 0 \end{cases} \qquad \begin{cases} x^2+y^2 > 1 \\ x^2+y^2 \leq 4 \\ y \neq -x \end{cases}$$



Use the method of Lagrange multipliers to find the minimum value of the function

$f(x, y, z) = x + y + z$

subject to the constraint $x^2 + y^2 + z^2 = 1$.

The constraint set is a sphere with center the origin and radius 1. Thus, it is a compact set.

$f(x, y, z)$ is a continuous function.

By Weierstrass's theorem, maximum and minimum exist.

$g(x, y, z) = x^2 + y^2 + z^2$

Qualification constraint: $\nabla g \neq \underline{0}$

$(2x, 2y, 2z) \neq (0, 0, 0)$

$(x, y, z) \neq (0, 0, 0)$

But $(0, 0, 0)$ does not belong to the constraint set. Thus, constraint qualification is always satisfied.

$$\mathcal{L}(x, y, z, \lambda) = x + y + z - \lambda(x^2 + y^2 + z^2 - 1)$$

$$\nabla \mathcal{L} = \underline{0}$$

$$\frac{\partial \mathcal{L}}{\partial x} = 0 : \quad 1 - 2\lambda x = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = 0 : \quad 1 - 2\lambda y = 0$$

$$\frac{\partial \mathcal{L}}{\partial z} = 0 : \quad 1 - 2\lambda z = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 : \quad x^2 + y^2 + z^2 = 1$$

Since it can't be $\lambda = 0$, we have:

$$\begin{cases} x = \frac{1}{2\lambda} \\ y = \frac{1}{2\lambda} \\ z = \frac{1}{2\lambda} \end{cases}$$

$$\left(\frac{1}{2\lambda}\right)^2 + \left(\frac{1}{2\lambda}\right)^2 + \left(\frac{1}{2\lambda}\right)^2 = 1$$

$$\frac{3}{4\lambda^2} = 1 \; , \quad \lambda^2 = \frac{3}{4} \; , \quad \lambda = \pm \frac{\sqrt{3}}{2}$$

$A\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ $\quad f(A) = \frac{3}{\sqrt{3}} = \sqrt{3}$ global maximum

$B\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$ $\quad f(B) = -\frac{3}{\sqrt{3}} = -\sqrt{3}$ global minimum