University of Macerata
Mathematical Methods for Economics and Finance

Exam

June 21, 2023

Surname:
Name:
Student number:

Please, save your work in a Matlab script entitled **YourSurname_YourStudentNumber.m** and send it to mauromaria.baldi@unimc.it by the end of the exam. If you prefer, you can make a script for each exercise (when needed) and save it as:

YourSurname_YourStudentNumber_ex_ExerciseNumber.m In this case, please send an email to the teacher by the end of the exam with all the scripts zipped into a single folder entitle **YourSurname_YourStudentNumber**.

Don't forget to write your surname, name and student number on each sheet you are submitting.

1. Consider an annuity of equal and yearly payments of amount R_1 whose first payment occurs at year 4 and whose last payment occurs at year 7. Then, consider a perpetuity of equal and yearly payments of amount R_2 whose first payment occurs at year 10. Let the rate of (compound) interest be i. Let $R_1 = 1000$, $R_2 = 200$, and $i = 10\%$:

   - Find the value $w(\mathbf{R}_1, 0)$ of the annuity at year 0
   - Find the value $w(\mathbf{R}_2, 0)$ of the perpetuity at year 0
   - Find the value $w(0)$ of the annuity together with the perpetuity at year 0
   - Find the value $w(7)$ of the annuity together with the perpetuity at year 7
   - (optional) Report your computations in Matlab.

   Hint: recall that

$$\sum_{k=0}^{n} x^k = \frac{1 - x^{n+1}}{1 - x}.$$

2. Sketch the domain of the function $f(x, y) = \sqrt{y - x^2}$ and find the gradient of f for a generic point (x, y) within the domain of f. Finally, plot the graph of f in Matlab.

3. Consider the following matrix:

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}.$$

   Find the eigenvalues of A. For each eigenvalue, find an associated eigenvector. Optional: find a comfirmation of your results in Matlab.

4. In Matlab:

   - Consider the matrix A of the previous exercise and store it into the matrix B
   - Set the element in position (1, 3) equal to 2 and store the new result in the matrix B
   - Set all the elements of the third row equal to 3 and store the new result in the matrix B
   - Delete the second row and the second column and store the new result in the matrix B
   - Let $g(x) = x^2 - 2x + \cos(x)$ and $h(x) = 10\sqrt{1 + \sin(x)}$, make a plot showing both functions in the same figure
   - Compute $g(B)$, $h(B)$, and $C = g(B) + h(B)^T$.

5. Solve the following problem:

$$\max_{(x,\, y) \in \Omega} f(x, y) = x^2 - xy + y,$$

   where

$$\Omega = \left\{(x, y) \in \mathbb{R}^2 : x + y \leq 4,\ x \geq 0,\ y \geq 0\right\}.$$

   Optional: can you prove the existence of a global maximum and minimum? If so, why? Can you find a global minimum?