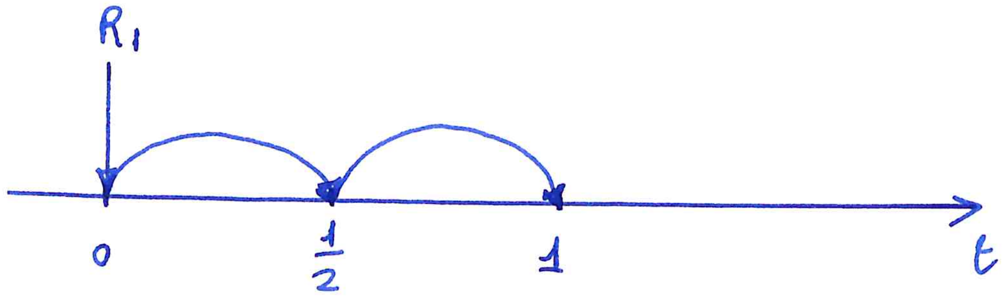
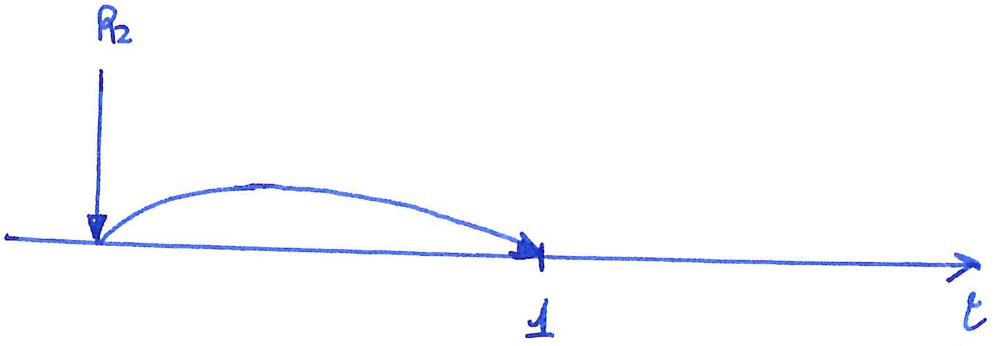
1)



2)



1

$$R_1 (1 + i \cdot 0.5)(1+i)^{0.5} = 1000 \text{ €}$$

$$R_1 = \frac{1000 \text{ €}}{(1 + 0.5i)\sqrt{1+i}} =$$

$$= \frac{1000 \text{ €}}{1.05 \sqrt{1.1}} \cong 908.06 \text{ €}$$

2

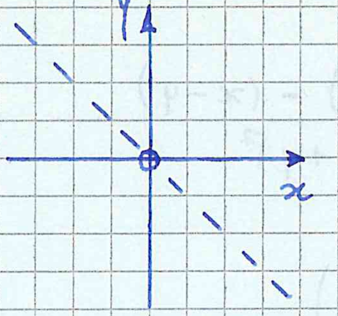
$$R_2 (1+i)^1 = 1000 \text{ €}$$

$$R_2 = \frac{1000 \text{ €}}{1+i} = \frac{1000 \text{ €}}{1.1} \cong 909.09 \text{ €}$$

Find the domain, the level curves and the gradient of

$$f(x, y) = \frac{x - y}{x + y}$$

Domain of f: $\quad x + y \neq 0, \quad y \neq -x$



Level curves:

$$\frac{x - y}{x + y} = c$$

If $c = 0$, $\quad \frac{x - y}{x + y} = 0; \quad x - y = 0; \quad y = x$

(without the point $(0, 0)$).

If $c \neq 0$:

$$x - y = c(x + y)$$
$$x - y = cx + cy$$
$$x - cx = y + cy$$
$$(1 - c)x = (1 + c)y$$

$$y = \frac{1 - c}{1 + c}x \quad \text{without the point } (0, 0)$$

If $c = -1$, $\quad \frac{x - y}{x + y} = -1; \quad x \cancel{- y} = -x \cancel{- y}$

$2x = 0; \quad x = 0$

Gradient :

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \frac{x-y}{x+y} = \frac{(x+y) - (x-y)}{(x+y)^2} =$$

$$= \frac{2y}{(x+y)^2}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \frac{x-y}{x+y} = \frac{-(x+y) - (x-y)}{(x+y)^2} = -\frac{2x}{(x+y)^2}$$

$$\nabla f = \left(\frac{2y}{(x+y)^2} , -\frac{2x}{(x+y)^2} \right)$$

Find and classify the critical points of the following function:

$f(x,y) = x^3 + y^3 - 3xy$

$\text{dom } f = \mathbb{R}^2$

$f'_x = 3x^2 - 3y \quad ; \quad f'_y = 3y^2 - 3x$

$\begin{cases} 3x^2 - 3y = 0 \\ 3y^2 - 3x = 0 \end{cases} ; \quad \begin{cases} x^2 - y = 0 \\ y^2 - x = 0 \end{cases} ;$

$\begin{cases} y = x^2 \\ x^4 - x = 0 \end{cases}$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$x(x-1)(x^2+x+1) = 0$

$x = 0 \lor x - 1 = 0 \lor x^2 + x + 1 = 0 \leftarrow \nexists\, x \in \mathbb{R}: x^2+x+1=0$

$\begin{cases} x = 0 \\ y = 0 \end{cases} , \quad \begin{cases} x = 1 \\ y = 1 \end{cases}$

$A(0,0) \quad , \quad B(1,1)$

$f''_{xx} = 6x \quad ; \quad f''_{xy} = -3 \quad ; \quad f''_{yx} = -3 \quad ; \quad f''_{yy} = 6y$

$\underline{\underline{H}}f = \begin{bmatrix} 6x & -3 \\ -3 & 6y \end{bmatrix}$

$\underline{\underline{H}}f(A) = \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix}$ $\quad \det \underline{\underline{H}}f(A) = -9 < 0$, then A is a saddle point

$\underline{\underline{H}}f(B) = \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}$ $\quad f''_{xx}(B) = 6 > 0 \; ; \; \det \underline{\underline{H}}f(B) = 27 > 0$, then $Hf(B) \succ 0$ so B is a local minimum.

Find the minimum distance from point $(0, 1)$ to the parabola $x^2 = 4y$

$$d = \sqrt{x^2 + (y-1)^2}$$

$$d^2 = x^2 + (y-1)^2$$

$$\min \quad x^2 + (y-1)^2$$

$$\text{s.t.:} \quad x^2 - 4y = 0$$

$$g(x,y) = x^2 - 4y$$

$\nabla g = (2x, -4)$ is always $\neq (0,0)$

$$\mathcal{L}(x,y,\lambda) = x^2 + (y-1)^2 - \lambda(x^2 - 4y)$$

$$\nabla \mathcal{L} = \underline{0}$$

$$\frac{\partial \mathcal{L}}{\partial x} = 0 \qquad 2x - 2\lambda x = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = 0 \qquad 2(y-1) + 4\lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \qquad -(x^2 - 4y) = 0$$

$$\begin{cases} (1-\lambda)x = 0 \\ y - 1 + 2\lambda = 0 \\ x^2 - 4y = 0 \end{cases}$$

$$\begin{cases} \lambda = 1 \\ y = -1 \\ x^2 = -4 \end{cases} \quad \text{impossible} \qquad \vee \qquad \begin{cases} x = 0 \\ \lambda = \frac{1}{2} \\ y = 0 \end{cases}$$

$$d_{min} = \sqrt{0^2 + (0-1)^2} = \sqrt{1} = \underline{1}$$