

University of Macerata
Mathematical Methods for Economics and Finance

Exam

July 12, 2023

Surname:

Name:

Student number:

Please, save your work in a Matlab script entitled **YourSurname_YourStudentNumber.m** and send it to mauromaria.baldi@unimc.it by the end of the exam. If you prefer, you can make a script for each exercise (when needed) and save it as:

YourSurname_YourStudentNumber_ex_ExerciseNumber.m In this case, please send an email to the teacher by the end of the exam with all the scripts zipped into a single folder entitle **YourSurname_YourStudentNumber**.

Don't forget to write your surname, name and student number on each sheet you are submitting.

1. You want to invest money today to collect 1000 euros in one year with $i = 10\%$. You have two options. Option 1) consists of investing an initial amount for six months with simple interest and then reinvest it for six more months with compound interest. Option 2) consists in making a single investment using a compound interest rule.
 - What should the initial amount of money investing using solution 1)?
 - What should the initial amount of money investing using solution 2)?
 - What is the most convenient solution?
2. In Matlab:
 - (a) Define \mathbf{v} as the vector with the first four positive natural numbers
 - (b) Define \mathbf{v}_1 as the first half of the vector \mathbf{v} and \mathbf{v}_2 as the second half of the vector \mathbf{v}_1
 - (c) Define \mathbf{A} as the matrix having the vectors \mathbf{v}_1 and \mathbf{v}_2 as rows

- (d) Redefine the matrix \mathbf{A} as follows:

$$\mathbf{A} := \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_2 \end{bmatrix},$$

where $\mathbf{0}$ is an appropriate matrix of zeros and \mathbf{I}_2 is the identity matrix of order 2

- (e) Set to -1 those elements in the matrix \mathbf{A} that are equal to 0
 - (f) Delete the second and third column from \mathbf{A}
 - (g) Delete the second and third row from \mathbf{A}
 - (h) Redefine \mathbf{A} as the resulting matrix after the previous operations and consider the functions $f(x) = \sqrt{1+x^2}$ and $g(x) = \ln(2 + \cos(x))$. Sketch in the same figure the graphs of f and g .
 - (i) Compute $\mathbf{B} = f(\mathbf{A})$, $\mathbf{C} = g(\mathbf{A})$ and $\mathbf{B}^T - \mathbf{C}^T$.
3. Find the domain, the level curves (without using Matlab), and the gradient of

$$f(x, y) = \frac{x-y}{x+y}.$$

After which, use Matlab to sketch in two separate figures the function f and the level curves. Optional: draw the function f with its level curves also in a vertical subplot.

4. Find and classify the critical points of the following function f :

$$f(x, y) = x^3 + y^3 - 3xy.$$

5. Use the Lagrange multipliers technique to find the minimum distance from point $(0, 1)$ to the parabola $x^2 = 4y$. Optional: is there a maximum distance from point $(0, 1)$ to the same parabola? Hint: sketch the problem graphically.