Determine the definiteness of the following symmetric matrix:

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 5 \\ 3 & 0 & 4 & 0 \\ 0 & 5 & 0 & 6 \end{bmatrix}$$

$$|A_1| = 1 > 0$$

$$|A_2| = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2 > 0$$

$$|A_3| = \begin{vmatrix} 1 & 0 & 3 \\ 0 & 2 & 0 \\ 3 & 0 & 4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} =$$

$$= 2(4-9) = 2 \cdot (-5) = -10 < 0$$

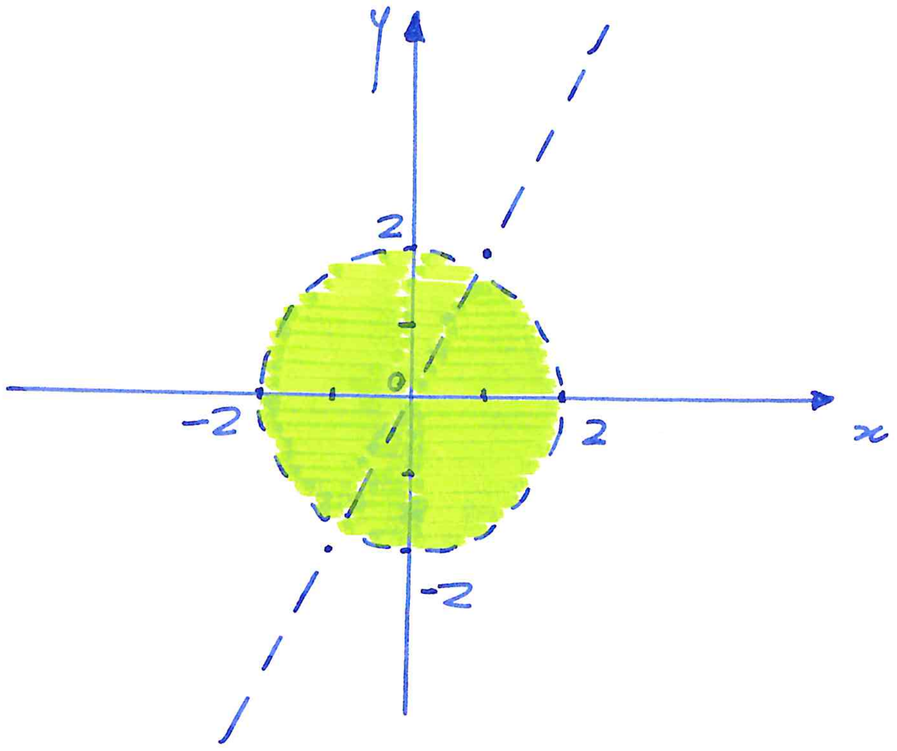
The matrix $\underline{\underline{A}}$ is indefinite

Sketch the domain of the following function:

$$z = \frac{\sqrt[3]{x+y}}{2x-y} + \ln\left(4 - x^2 - y^2\right)$$

$2x - y \neq 0$; $y \neq 2x$

$4 - x^2 - y^2 > 0$; $x^2 + y^2 < 4$



Given the following function defined on $\mathbb{R}^2$, find the critical points and classify these as local max, local min, saddle point or "can't tell"

$$z = xy^2 + x^3y - xy$$

$$f'_x = y^2 + 3x^2y - y = 0$$

$$f'_y = 2xy + x^3 - x = 0$$

$$\begin{cases} y^2 + 3x^2y - y = 0 \\ 2xy + x^3 - x = 0 \end{cases}$$

$$\begin{cases} y(y + 3x^2 - 1) = 0 \\ x(2y + x^2 - 1) = 0 \end{cases}$$

$$\text{I} \quad \begin{cases} y = 0 \\ x = 0 \end{cases} \qquad \text{II} \quad \begin{cases} y = 0 \\ 2y + x^2 - 1 = 0 \end{cases}$$

$$\text{III} \quad \begin{cases} y + 3x^2 - 1 = 0 \\ x = 0 \end{cases} \qquad \text{IV} \quad \begin{cases} y + 3x^2 - 1 = 0 \\ 2y + x^2 - 1 = 0 \end{cases}$$

Case I: $\quad O(0,0)$

Case II: $\begin{cases} y = 0 \\ 2y + x^2 - 1 = 0 \end{cases}$

$\begin{cases} y = 0 \\ x^2 - 1 = 0 \end{cases} \qquad \begin{cases} y = 0 \\ x^2 = 1 \end{cases} \qquad \begin{cases} y = 0 \\ x = \pm 1 \end{cases}$

$A(1, 0)$; $\quad B(-1, 0)$

Case III: $\begin{cases} y + 3x^2 - 1 = 0 \\ x = 0 \end{cases} \qquad \begin{cases} y - 1 = 0 \\ x = 0 \end{cases} \qquad \begin{cases} y = 1 \\ x = 0 \end{cases}$

$C(0, 1)$

Case IV: $\begin{cases} y + 3x^2 - 1 = 0 \\ 2y + x^2 - 1 = 0 \end{cases}$

$\begin{cases} y = -3x^2 + 1 \\ 2(-3x^2 + 1) + x^2 - 1 = 0 \end{cases} \qquad \begin{cases} y = -3x^2 + 1 \\ -6x^2 + 2 + x^2 - 1 = 0 \end{cases}$

$\begin{cases} y = -3x^2 + 1 \\ -5x^2 + 1 = 0 \end{cases} \qquad \begin{cases} y = -3x^2 + 1 \\ 5x^2 = 1 \end{cases} \qquad \begin{cases} y = -3x^2 + 1 \\ x^2 = \frac{1}{5} \end{cases}$

$\begin{cases} y = -\frac{3}{5} + 1 \\ x = \pm \frac{1}{\sqrt{5}} \end{cases} \qquad \begin{cases} y = \frac{2}{5} \\ x = \pm \frac{1}{\sqrt{5}} \end{cases}$

$D\left(-\frac{1}{\sqrt{5}}, \frac{2}{5}\right)$; $\quad E\left(\frac{1}{\sqrt{5}}, \frac{2}{5}\right)$

Hessian test

$f'_x = y^2 + 3x^2 y - y$

$f''_{xx} = 6xy \qquad f''_{xy} = 2y + 3x^2 - 1$

$f'_y = 2xy + x^3 - x$

$f''_{yx} = 2y + 3x^2 - 1$

$f''_{yy} = 2x$

$$\underline{\underline{H}}_f(x,y) = \begin{bmatrix} 6xy & 2y + 3x^2 - 1 \\ 2y + 3x^2 - 1 & 2x \end{bmatrix}$$

$$\underline{\underline{H}}_f(0) = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

The Hessian test fails but the Hessian matrix is indefinite; saddle point (because its eigenvalues are $+1$ and -1)

$$\underline{\underline{H}}_f(A) = \underline{\underline{H}}_f(1, 0) = \begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$$ indefinite; saddle point

In fact: $-\lambda(2-\lambda) - 4 = 0$; $\lambda(\lambda - 2) - 4 = 0$; $\lambda^2 - 2\lambda - 4 = 0$

$\lambda = 1 \pm \sqrt{5}$

$$\underline{\underline{H}}_f(B) = \underline{\underline{H}}_f(-1, 0) = \begin{bmatrix} 0 & 2 \\ 2 & -2 \end{bmatrix}$$ indefinite; saddle point

In fact: $-\lambda(-2-\lambda) - 4 = 0$; $\lambda^2 + 2\lambda - 4 = 0$

$\lambda = -1 \pm \sqrt{5}$

$$\underline{\underline{H}}f(C) = \underline{\underline{H}}f(0,1) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$ indefinite;

The eigenvalues are ± 1

$$\underline{\underline{H}}f(D) = \underline{\underline{H}}f\left(-\frac{1}{\sqrt{5}}, \frac{2}{5}\right) =$$

$$= \begin{bmatrix} -\frac{12}{5\sqrt{5}} & \frac{4}{5}+\frac{3}{5}-1 \\ \frac{4}{5}+\frac{3}{5}-1 & -\frac{2}{\sqrt{5}} \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{12}{5\sqrt{5}} & \frac{2}{5} \\ \frac{2}{5} & -\frac{2}{\sqrt{5}} \end{bmatrix}$$

$$|H_1| = -\frac{12}{5\sqrt{5}} < 0$$

$$|H_2| = \begin{vmatrix} -\frac{12}{5\sqrt{5}} & \frac{2}{5} \\ \frac{2}{5} & -\frac{2}{\sqrt{5}} \end{vmatrix} = \frac{24}{25} - \frac{4}{25} = \frac{20}{25} = \frac{4}{5} > 0$$

$$\underline{\underline{H}}f(D) < 0$$ D is a local maximum

$$\underline{\underline{H}}_f(E) = \underline{\underline{H}}_f\left(\frac{1}{\sqrt{5}}, \frac{2}{5}\right) =$$

$$= \begin{bmatrix} \frac{12}{5\sqrt{5}} & \frac{2}{5} \\ \frac{2}{5} & \frac{2}{\sqrt{5}} \end{bmatrix}$$

$$|H_1| = \frac{12}{5\sqrt{5}} > 0$$

$$|H_2| = \begin{vmatrix} \frac{12}{5\sqrt{5}} & \frac{2}{5} \\ \frac{2}{5} & \frac{2}{\sqrt{5}} \end{vmatrix} =$$

$$= \frac{24}{25} - \frac{4}{25} = \frac{20}{25} = \frac{4}{5} > 0$$

$$\underline{\underline{H}}_f(E) \succ 0$$

E is a local minimum

Determine the annual IRR for the following project: $\{(-100, 30, 80);\ (0, 2, 4)\}$ both analitically and by using Matlab

$$-100 + 30(1+i)^{-2} + 80(1+i)^{-4} = 0$$

$$x := (1+i)^{-2}$$

$$80x^2 + 30x - 100 = 0$$

$$8x^2 + 3x - 10 = 0$$

$$x = \frac{-3 \pm \sqrt{9 + 4 \cdot 8 \cdot 10}}{16} = \frac{-3 \pm \sqrt{329}}{16} \cong$$

0.9461

-1.3211 not acceptable

$$x = 0.9461$$

$$(1+i)^{-2} = 0.9461$$

$$(1+i)^{2} = \frac{1}{0.9461}$$

$$1 + i = \frac{1}{\sqrt{0.9461}}$$

$$i = \frac{1}{\sqrt{0.9461}} - 1 \approx 0.0281$$