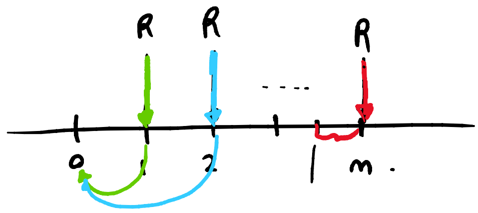
giovedì 23 febbraio 2023 08:51

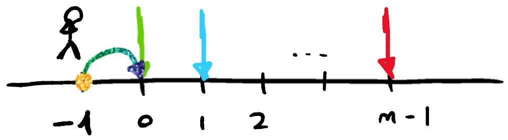
ordinary annuity



$$w(R,0) = R\,\underbrace{\frac{1-(1+i)^{-m}}{i}}_{a_{m|i}}$$

$a_{m|i}$ a angle m at i

↓ annuity due

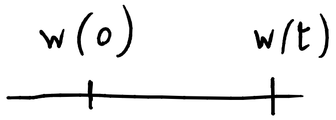


$$w(R,-1) = R\, a_{\overline{m}|i}$$

$$w(R,0) = R\,\underbrace{\frac{1-(1+i)^{-m}}{i}\,(1+i)}_{\ddot{a}_{m|i}}$$

$\ddot{a}_{m|i}$ a dots angle m at i

ordinary annuity



$$w(t) = w(0)(1+i)^t$$

$$w(R,0)$$

$$w(R,m) =$$

$$= w(R,0)(1+i)^m = R\,a_{\overline{m}|i}(1+i)^m =$$

$$= R\,\frac{1-(1+i)^{-m}}{i}\cdot(1+i)^m =$$

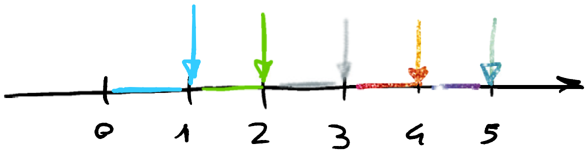
$$= R\,\underbrace{\frac{(1+i)^m-1}{i}}_{s_{\overline{m}|i}}$$

$$A\cdot(B+C) =$$

$$= A\cdot B + A\cdot C$$

$$(B+C)\cdot A = B\cdot A + C\cdot A$$

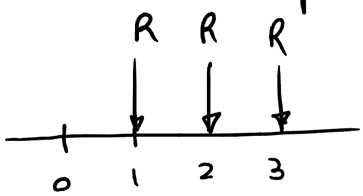
EX14: Calculate the present value of an ordinary annuity of amount \$100 paid annually for 5 years at the rate of interest of 9%. Also calculate its future value at time 5.

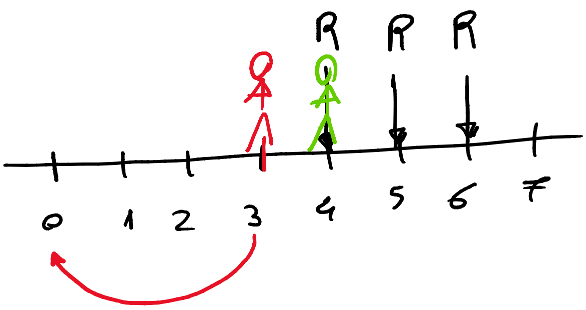


$$W(R,0) = R \underbrace{\frac{1-(1+i)^{-m}}{i}} \simeq 366{,}97 \$$$

$a_{\overline{m}|i}$ a angle m

DEFERRED ANNUITY: it is an annuity that starts at some time in the future.





deferred ordinary annuity starting at time 3

or

annuity due starting at time 4

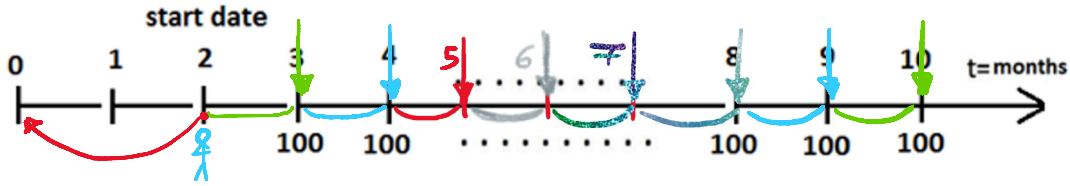
$$W(R,3) = R\, a_{\overline{m}|i}$$

$$W(R,0) = W(R,3)\,(1+i)^{-3}$$

EX16: Consider a monthly annuity-immediate of 100 euro starting at 2 months and ending at 10 months. Determine its present value given i=0.1.

i_{12}

Graphic representation:



$$i_{12} = (1+i)^{\frac{1}{12}} - 1 = 0.008$$

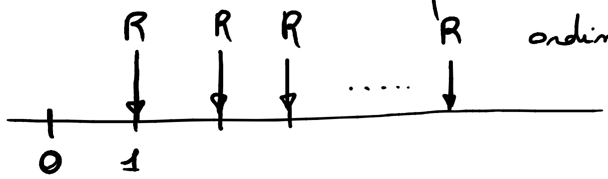
$m = 8$

$$W(R,2) = R\,\frac{1-(1+i_{12})^{-m}}{i_{12}} \cong 772 \text{ €}$$

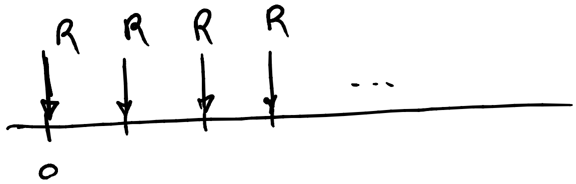
$$W(R,0) = W(R,2)\,(1+i_{12})^{-2} \cong 759{,}87 \text{ €}$$

PERPETUITY

It is an annuity that never ends



ordinary perpetuity



perpetuity due

m payments

$m \rightarrow +\infty$

ordinary annuity

$$W(R,0) = R\,a_{\overline{m}|i} = R\,\frac{1-(1+i)^{-m}}{i}$$

ordinary perpetuity

$$W(R,0) = \lim_{m \to +\infty} R\,a_{\overline{m}|i}$$

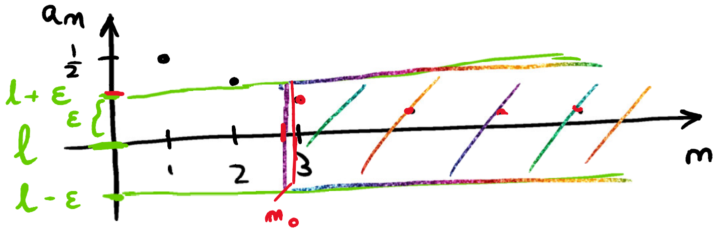
Sequence $\quad \{\underline{a_n}\}_{n=1}^{+\infty}$

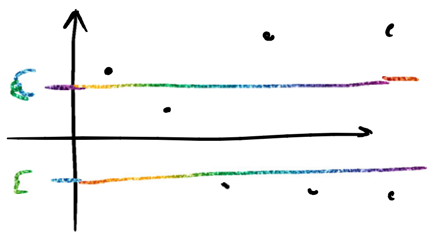
$$y = f(x)$$

$$a_n = \frac{1}{2\sqrt{n}}$$

$n = 1 \qquad a_1 = \frac{1}{2\sqrt{1}} = \frac{1}{2}$

$n = 2 \qquad a_2 = \frac{1}{2\sqrt{2}}$





$$\lim_{n \to +\infty} a_n = l$$

small - enough value

$\forall \; \varepsilon > 0 \quad \exists \; n_0 :$

$\forall \; n > n_0 , \qquad \underline{|a_n - l| < \varepsilon}$

is equivalent to

$$-\varepsilon < a_n - l < \varepsilon$$

$$l - \varepsilon < a_n < l + \varepsilon$$

ordinary perpetuity

$$W(R,\infty) = \lim_{n \to +\infty} R\, a_{\overline{n}|i} =$$

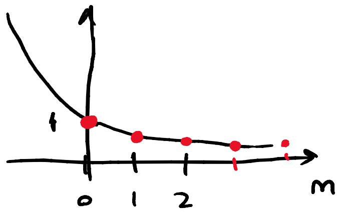
$$= \lim_{n \to +\infty} R \cdot \frac{1 - (1+i)^{-n}}{i} =$$

$$= \frac{R}{} \lim_{n \to +\infty} \left[1 - (1+i)^{-n}\right] =$$

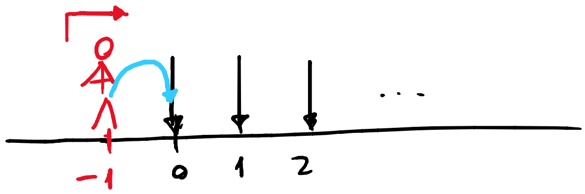
$$= \frac{R}{i} \lim_{m \to +\infty} \left[1 - (1+i)^{-m} \right] =$$

$$= \frac{R}{i} \left[\underbrace{\lim_{m \to +\infty} 1} - \lim_{m \to +\infty} \frac{1}{(1+i)^m} \right] =$$

$$= \frac{R}{i} \left[1 - \lim_{m \to +\infty} \underbrace{\left(\frac{1}{1+i} \right)^m} \right] = \underbrace{R \frac{1}{i}}_{a_{\overline{\infty}|i}}$$



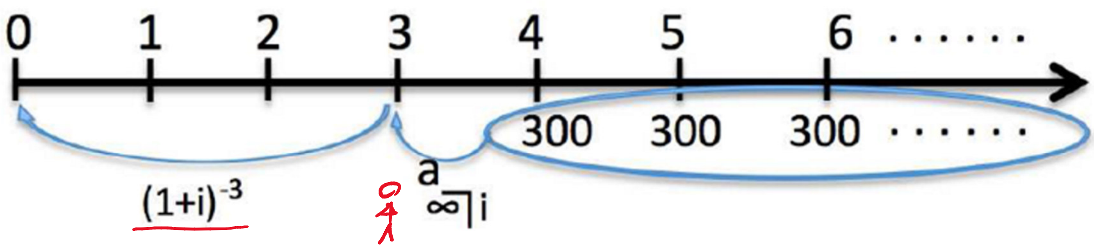
$W(R, 0)$



$$W(R, -1) = R \frac{1}{i}$$

$$W(R, 0) = \underbrace{R \frac{1}{i} (1+i)}_{\ddot{a}_{\overline{\infty}|i}}$$

EX20: Consider an annual perpetuity of 300 euro, the first payment is made at the end of the third year and the interest rate is 0.06. The present value is obtained as follows.



$\hookrightarrow R \cdot \frac{1}{i}$

$W(R, 0) = R \frac{1}{i} (1+i)^{-3} =$

$$W(R,0) = R\,\frac{1}{i}\,(1+i)^{-3} =$$

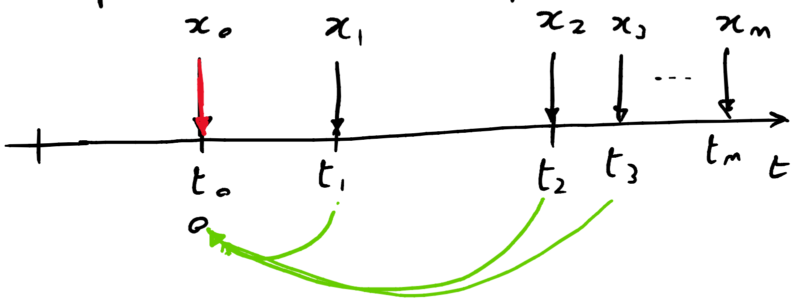
$$= \frac{300€\,(1+0{,}06)^{-3}}{0{,}06} \cong 4198\ €$$

Internal rate of return (IRR)

Cash flow:

$$CF = \{(x_0, x_1, \dots, x_m);\ (t_0, t_1, \dots, t_m)\}$$

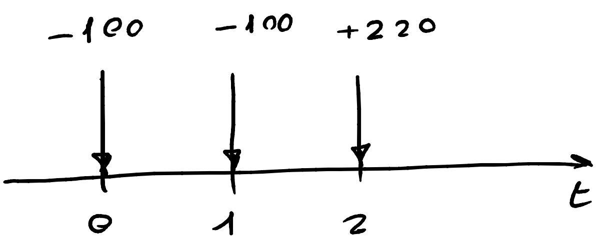
It is the rate of interest such that the present value of the cash flow is zero



$$W(0) = x_0 + x_1(1+i)^{-t_1} + x_2(1+i)^{-t_2} + \dots + x_m(1+i)^{-t_m}$$

EX22: Determine the IRR of the following financial project:

$$\text{L'OF} = \{(-100, -100, +220); (0, 1, 2)\}$$



$$W(0) = -100 - 100(1+i)^{-1} + 220(1+i)^{-2}$$

$^{-1}$

$$W(0) = -100 - 100(1+i)^{-1} + 220(1+i)^{-2}$$

$$x := (1+i)^{-1}$$

$$0 = -100 - 100x + 220x^2$$

$$220x^2 - 100x - 100 = 0$$

$$2{,}2x^2 - x - 1 = 0$$

$$x = \frac{1}{1+i} = -0.48$$

not acettable

$$x = \frac{1}{1+i} = 0{,}9387$$

$$1+i = \frac{1}{0{,}9387}$$

$$i = \frac{1}{0{,}9387} - 1 \cong 0{,}065$$

$$\boxed{-1}$$

c

array1 =

5	7	8
1	2	3