

# FUNCTION

martedì 7 marzo 2023 08:51

$$\mathbb{R}^m \longrightarrow \mathbb{R}$$

## CARTESIAN PRODUCT

$$A = \{\text{Carl}, \text{Anne}, \text{Elizabeth}\}$$

$$B = \{\text{sushi}, \text{pizza}\}$$

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

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such that

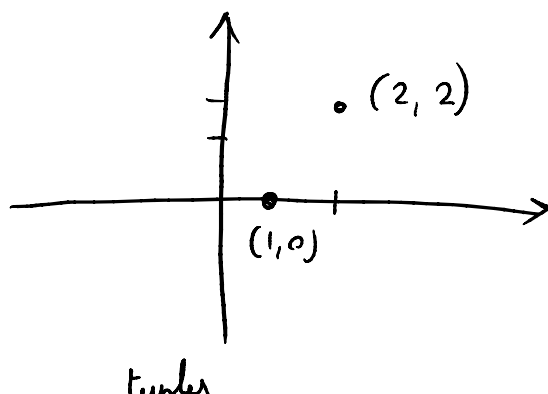
$$A \times B = \{(\text{Carl}, \text{sushi}), (\text{Carl}, \text{pizza}), (\text{Anne}, \text{sushi}), \\ (\text{Anne}, \text{pizza}), (\text{Elizabeth}, \text{sushi}), (\text{Elizabeth}, \text{pizza})\}$$

$$A \times A = A^2$$

$$A \times A \times A = A^3 = \{(\text{Carl}, \text{Anne}, \text{Elizabeth}), \dots\}$$

$\mathbb{R}$

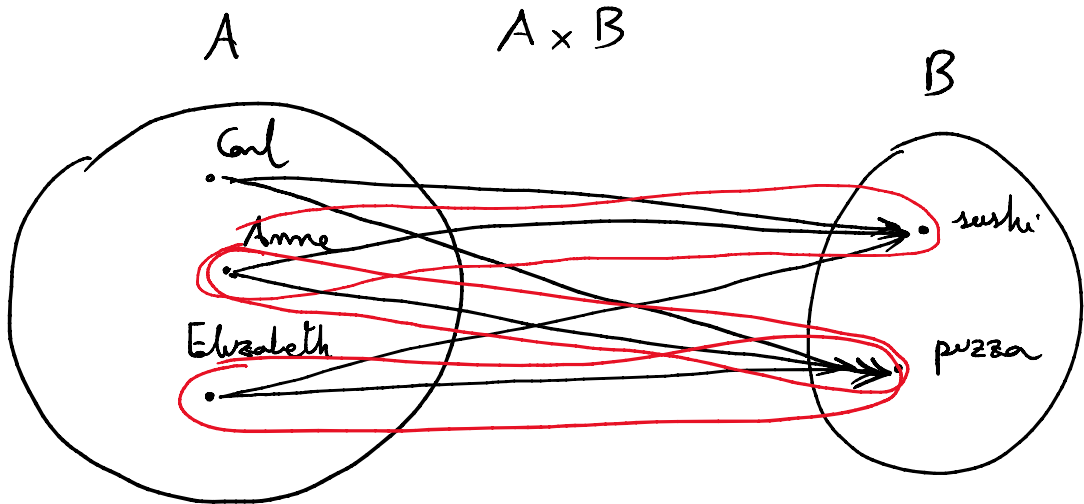
$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{\dots, (1, 0), (2, 2), \dots\}$$



$$\mathbb{R}^n = \left\{ \underbrace{(3, 2, 0, \dots, 1)}_{n \text{ elements}} \right\}$$

tuples

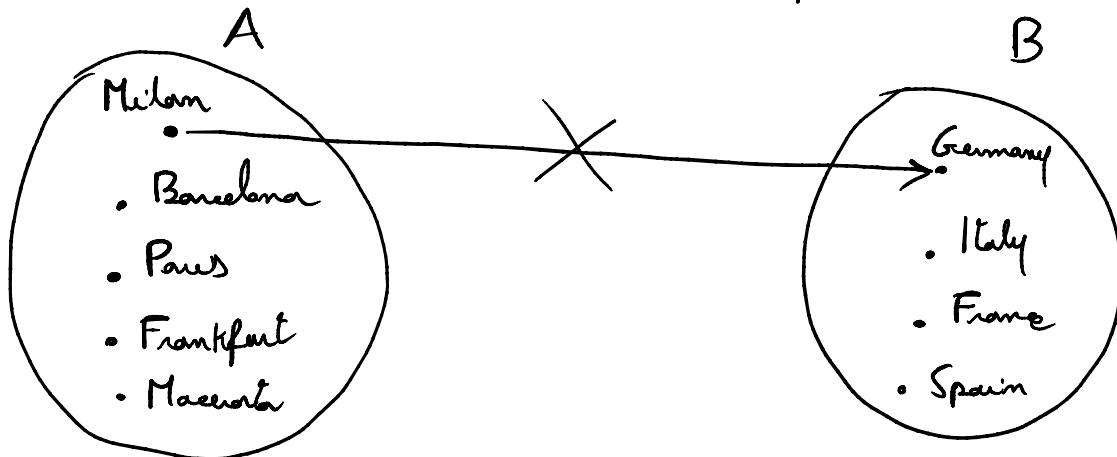
this is a vector



This is a subset of the cartesian product  
A relation is a subset of the cartesian product

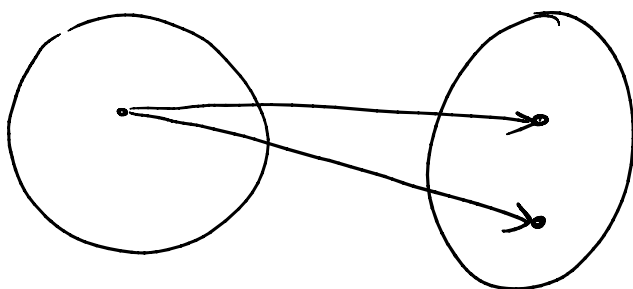
$$A = \{x \mid x \text{ is a city of Europe}\}$$

$$B = \{x \mid x \text{ is a states of Europe}\}$$



$p$

$$\mathcal{R} = \{ \underline{(x, y)} : x \text{ is a city of } y \}$$



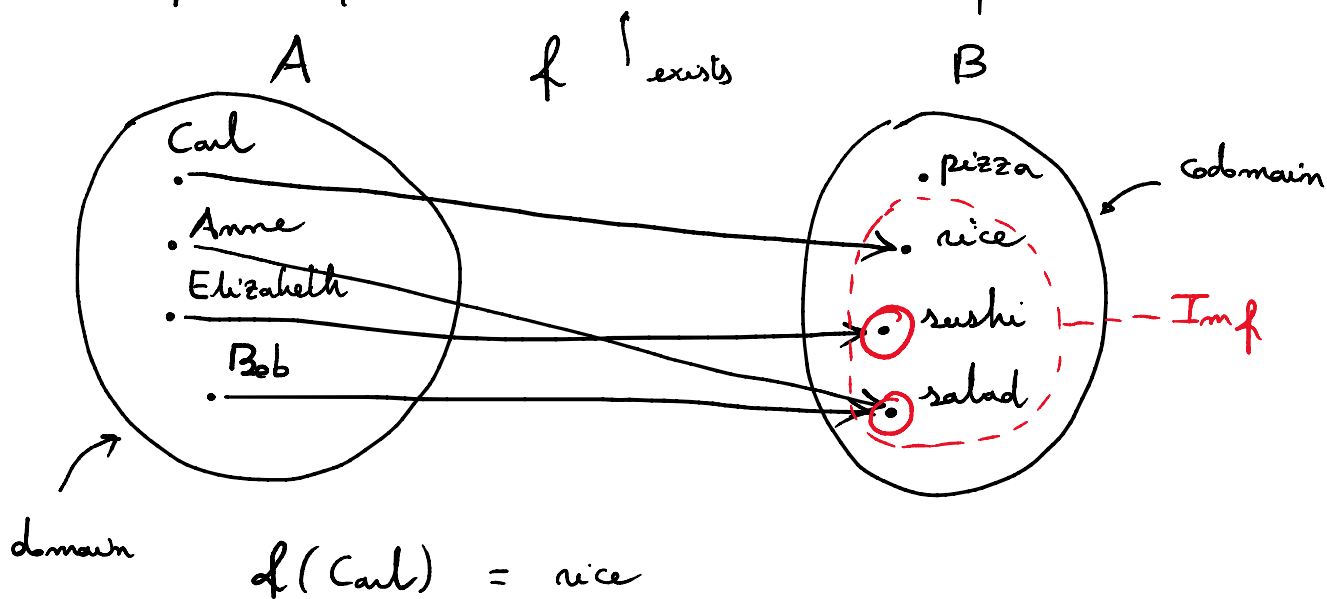
A function  $f: A \rightarrow B$  is a rule (or law) that assigns to each element  $x \in A$  one and only one element in  $B$ .

$A$ : is the domain

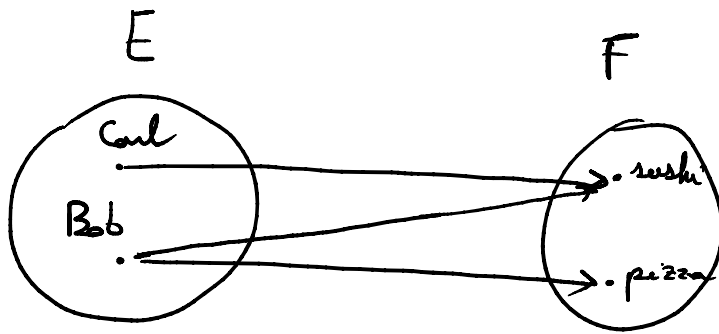
$B$ : is the codomain

The rule is denoted by  $y = f(x)$

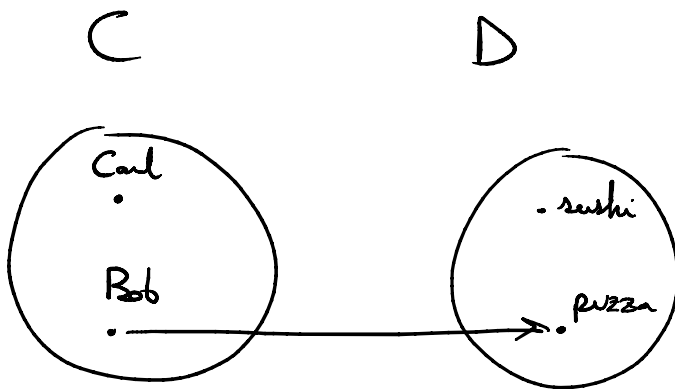
$$\text{Im} f = \{ y \in B \mid \exists \underline{x} \in A : y = f(x) \}$$



$$\text{Im} f = \{ \text{salad, sushi, rice} \}$$



This is not a function because Bob is eating more than one dish. This is a relation



This is not a function because Carl is not eating.

A function  $f: A \subseteq \mathbb{R} \rightarrow \mathbb{R}$  is a rule (or law) that assigns to each element  $x \in A$  one and only one element in  $\mathbb{R}$ .

$$y = x^2 - 2x + 1$$

$$u = U(x) \quad \text{utility function}$$

A function  $f: A \subseteq \mathbb{R}^m \rightarrow \mathbb{R}$  is a rule (or law) that assigns to each element  $x \in A$  one and only one element in  $\mathbb{R}$ .

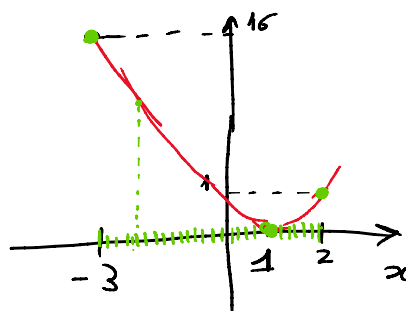
A function  $f: A \subseteq \mathbb{R} \rightarrow \mathbb{R}$  is a rule (or law) that assigns to each element  $x \in A$  one and only one element in  $\mathbb{R}$ .

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}$$
$$f(x, y) = x^2 + y^2$$

$$x = [-3, 1, 2]$$

$$f(x) = \underline{x^2} - 2x + 1$$

$$y = [(-3)^2 - 2(-3) + 1, 1^2 - 2 \cdot 1 + 1, 2^2 - 2 \cdot 2 + 1] =$$
$$= [16, 0, 1]$$



$$y = x \sin(x)$$

$$y = x \log(1+x)$$