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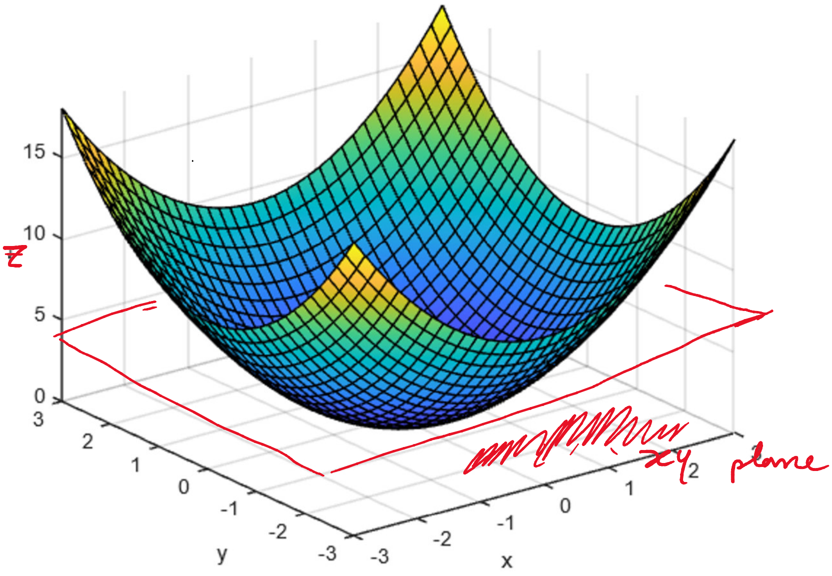
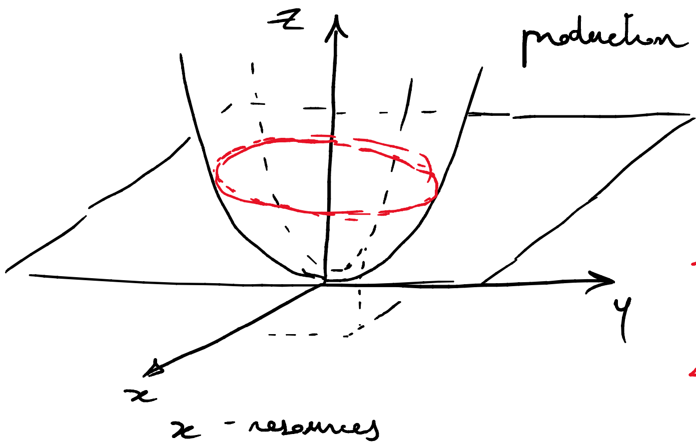


level curves

production function



all the points in the circle are at the same height (level) i.e., the value of the function is the same

locus = place

$$z = f(x, y)$$

you set z to be constant, say to a value k

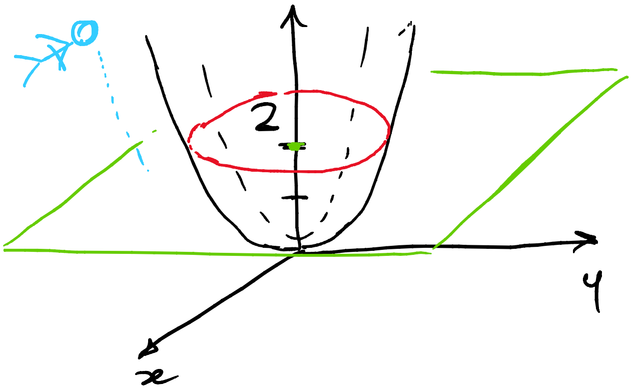
$$k = f(x, y)$$

All the points satisfying this equation are at the same level k.

$$z = x^2 + y^2$$

$z = x^2 + y^2$

we set z constant to a level 2



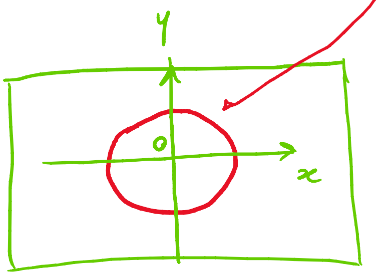
$z = x^2 + y^2$

$2 = x^2 + y^2$

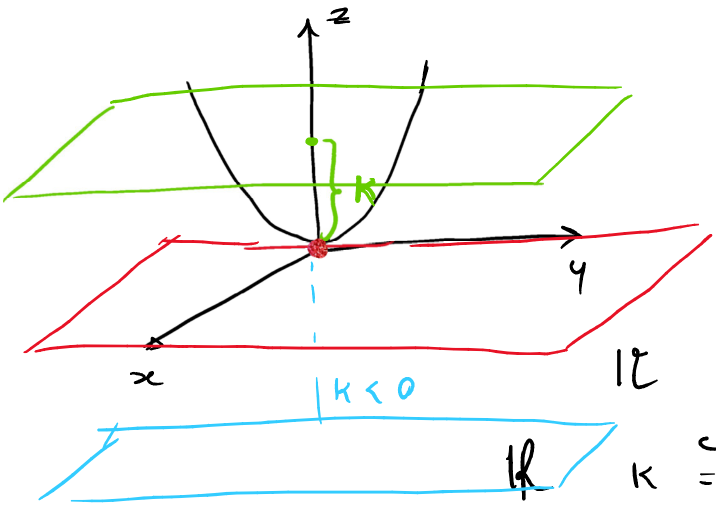
$x^2 + y^2 = 2$

$x^2 + y^2 = (\sqrt{2})^2$

circle center $(0,0)$, radius $\sqrt{2}$



In general we are cutting the surface at a level k



$z = x^2 + y^2$

$k = x^2 + y^2$

$x^2 + y^2 = k$

It is a circle if $k > 0$

$x^2 + y^2 = (\sqrt{k})^2$

center $(0,0)$ radius $\sqrt{k}$

If $k = 0$ it is a point

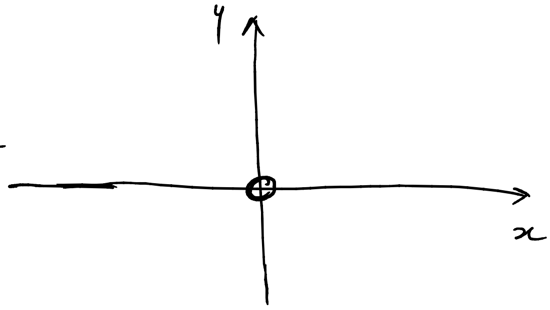
(degenerate circle of radius 0)

If $k < 0$ no intersections

(no level curves)

6. $z = e^{xy}$

$e^{0 \cdot 0} = e^{0} = 1$



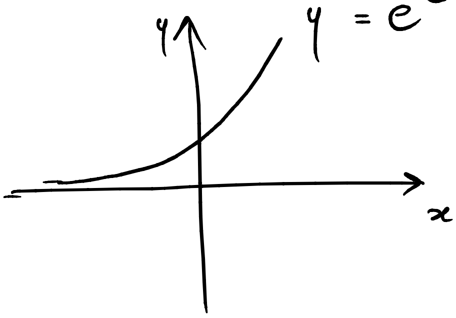
$e^{1 \cdot 2} = e^{2}$

$x = 1 \qquad y = -2$

domain: $\mathbb{R}^2$

$e^{-2} = \dfrac{1}{e^{2}}$

$y = e^{x}$



$z = e^{xy}$

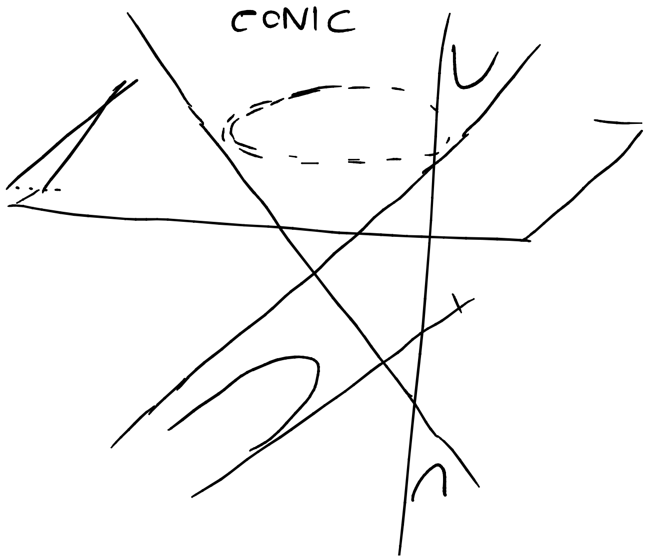
$k = e^{xy} \qquad k > 0$

$\ln k = \ln e^{xy}$

$\ln k = xy$

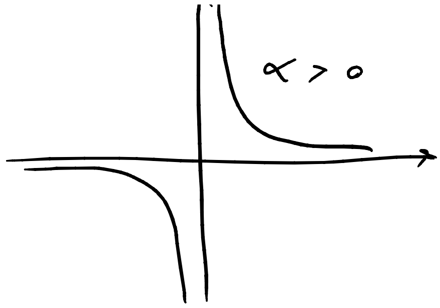
$xy = \ln k$

$xy =$



$xy = \alpha$

$\alpha > 0$



$$x = \log_a \overset{b > 0}{b}$$

3. $z = \sqrt{\ln(x-y)}$

$$\boxed{x - y > 0}$$

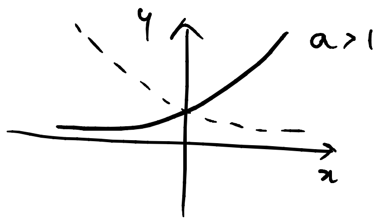
The logarithm $\log_a b$ is that number x that satisfies the equation

$$\underbrace{a^x}_{} = \underbrace{b}_{} \qquad (1)$$

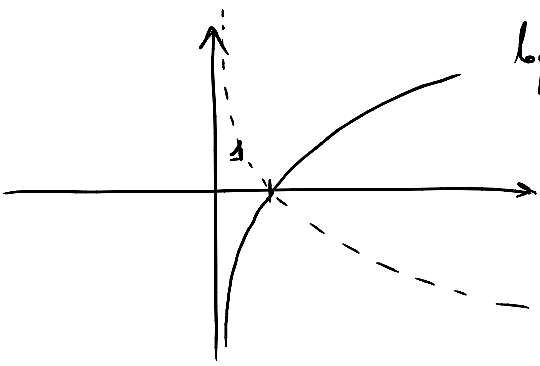
$a > 1$ or

$0 < a < 1$

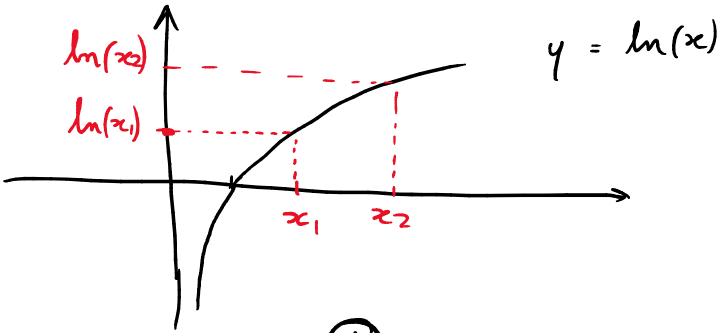
On the first side of equation (1), we have an exponential function that is always positive

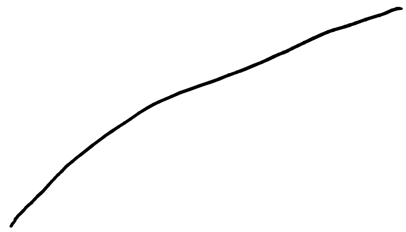


Thus, b has to be positive



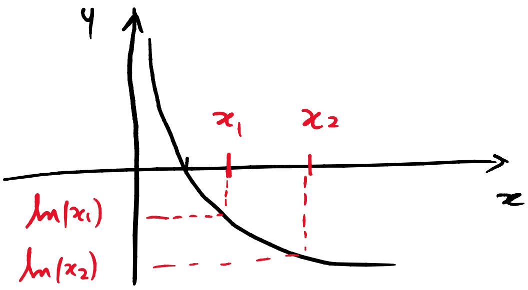
$$\begin{cases} x - y > 0 \\ \ln(x-y) \geqslant 0 \end{cases}$$





(*)

$\underline{x_2} > \underline{x_1} \iff \underline{\ln(x_2)} > \underline{\ln(x_1)}$ increasing monotonic function



$a \in (0,1)$

$x_1 < x_1 \iff \ln(x_1) > \ln(x_2)$ decreasing monotonic function

$\ln(x-y) \geqslant 0$

$e > 1$

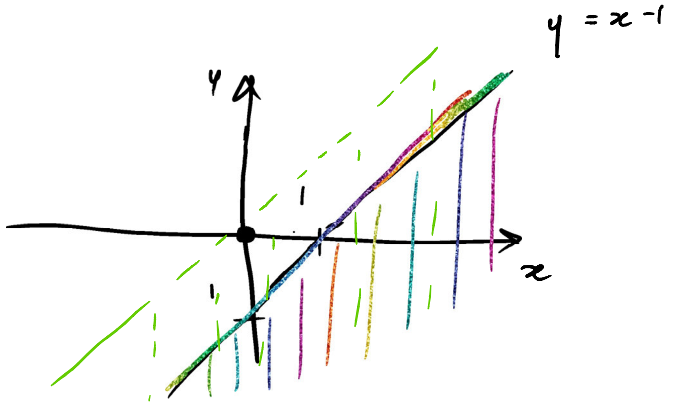
$\ln(x-y) \geqslant 0$

$\ln(x-y) \geqslant \ln 1$ (*)

$x - y \geqslant 1$

$$\begin{cases} x - y > 0 \\ x - y \geqslant 1 \end{cases}$$

$$\begin{cases} \ldots \\ x - y \geq 1 \end{cases}$$



$x - y \geq 1$

$y \leq x - 1$

$0 \leq 0 - 1$

$0 \leq -1$ NO