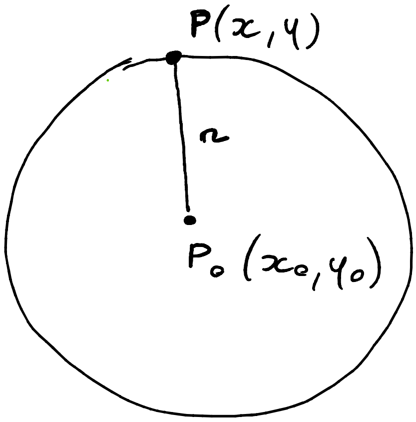
giovedì 16 marzo 2023 08:54

INDIFFERENCE CURVES

If $z = f(x, y)$ is a utility function then the level curves are said INDIFFERENCE CURVES. They represent the combinations of consumed quantities of each good corresponding to the same utility.

ISOQUANTS

If $z = f(x, y)$ is a production function then the level curves are said ISOQUANTS. They represent the combinations of quantities of each production factor corresponding to the same production level.



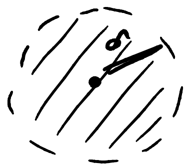
$$d(P, P_0) = r \qquad r \geq 0$$

$r = 0$ degenerate case

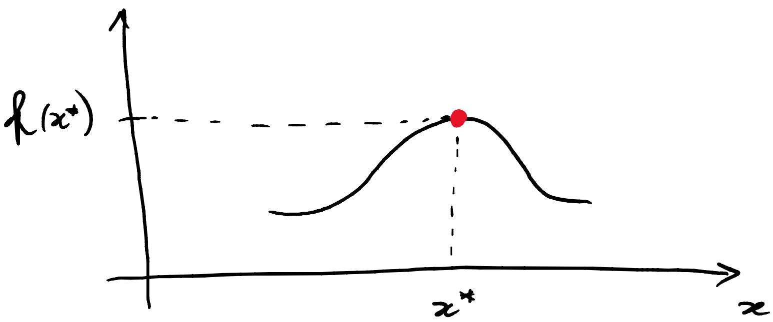
$$\sqrt{(x-x_0)^2 + (y-y_0)^2} = r$$

$$(x-x_0)^2 + (y-y_0)^2 = r^2$$

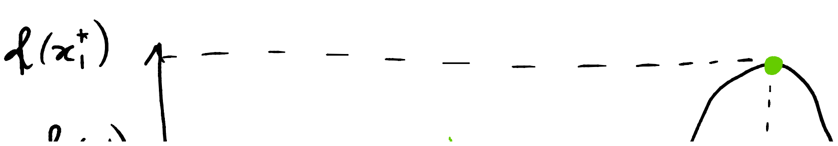
$$(x-x_0)^2 + (y-y_0)^2 < r^2$$



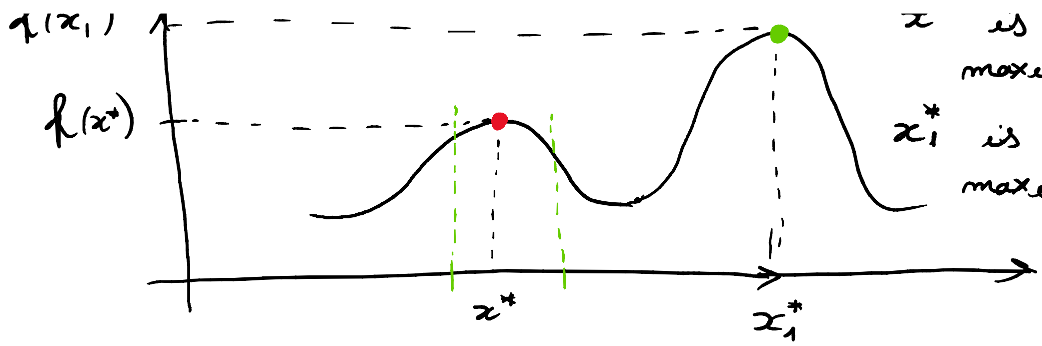
$$\sqrt{(x-x_0)^2 + (y-y_0)^2} < \delta$$



x^* is a global maximum

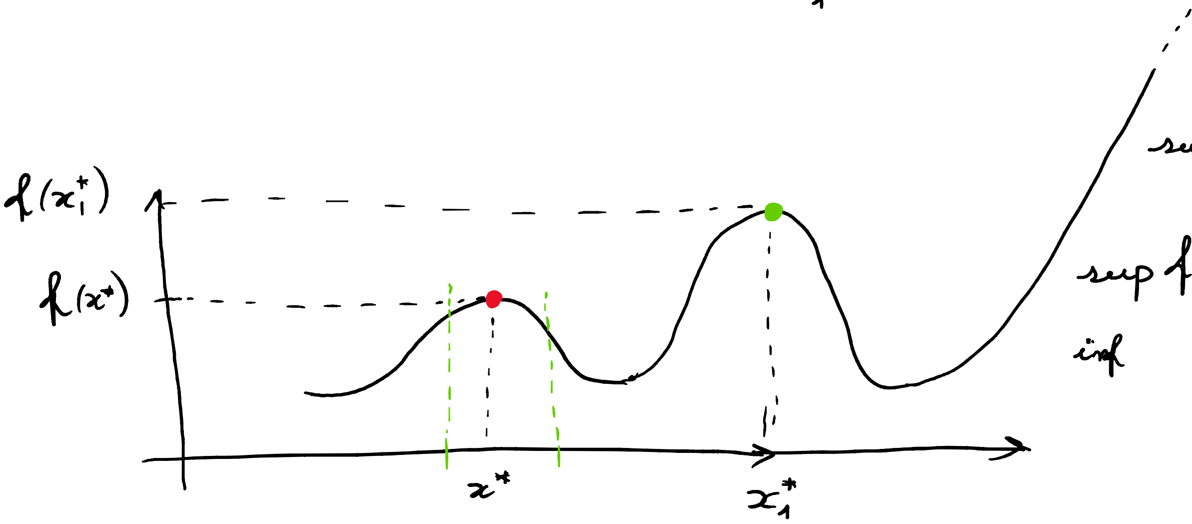


x^* is a local maximum



x is a local maximum

x_1^* is a global maximum



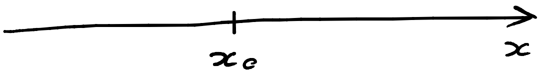
supremum

$\sup f(x) = +\infty$

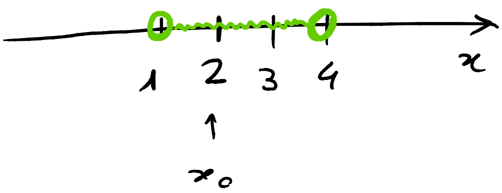
inf

x^* and x_1^* are local maxima

$\sup f(x) = +\infty$



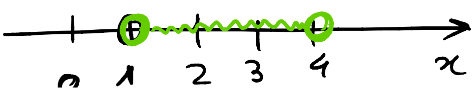
A neighborhood of x_0 is an open interval containing x_0



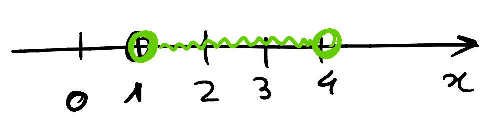
$(1, 4)$ or $]1, 4[$,

$1 < x < 4$

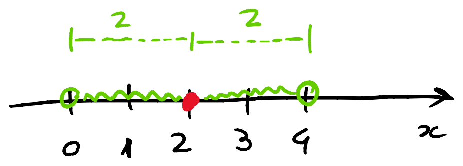
This is a neighborhood of 2



This is also a neighborhood of 2

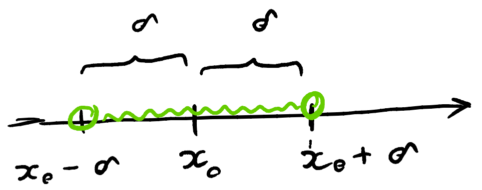


This is also a neighborhood of 2



This is a symmetric neighborhood of 2

$0 < x < 4$

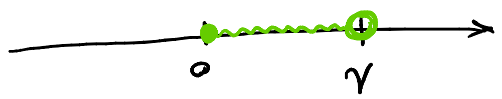


$$x_0 - \delta < x < x_0 + \delta$$

$$|A| = \begin{cases} A & \text{if } A \geq 0 \\ -A & \text{if } A < 0 \end{cases}$$

$|x| < r$ $\quad$ r in this case, cannot be zero or negative

if $x \geq 0$ $\quad$ $|x|$ becomes x so the inequality becomes $x < r$



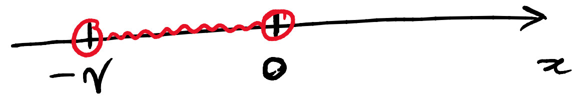
if $x < 0$ then $|x| = -x$ and the inequality becomes

$$|x| < r$$

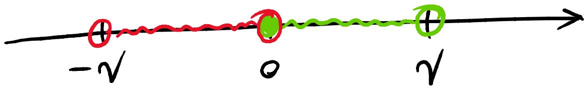
$$-x < r$$

$$(-1)(-x) > r(-1)$$

$$x > -r$$



Putting all together, we have



but this is a symmetric neighborhood of 0

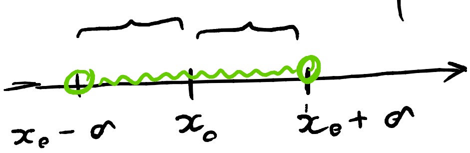
so the inequality $|x| < r$ is equivalent to $-r < x < r$

This is valid in general:

$$|g(x)| \leq B$$

$$-B \leq g(x) \leq B$$

σ σ : ray (radius) of the interval



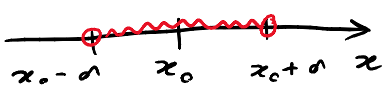
$$x_0 - \sigma < x < x_0 + \sigma$$

$$\rightarrow \cancel{x_0} - \sigma - \cancel{x_0} < x - x_0 < \cancel{x_0} + \sigma - \cancel{x_0}$$

$$\rightarrow -\sigma < x - x_0 < \sigma$$

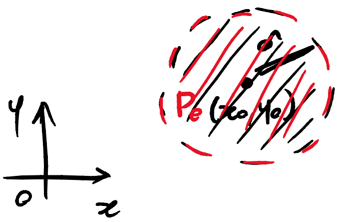
$$\rightarrow |x - x_0| < \sigma$$

$$|x - x_0| = \sqrt{(x - x_0)^2}$$

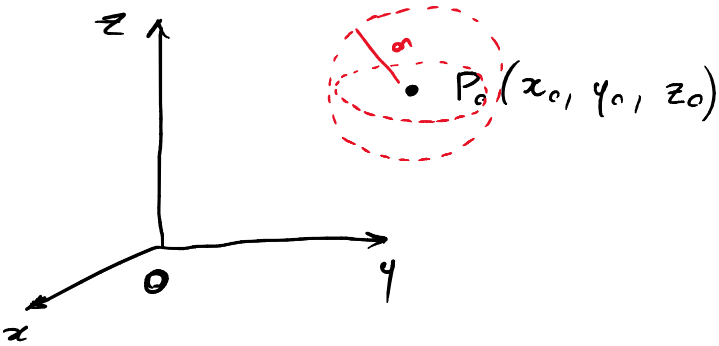


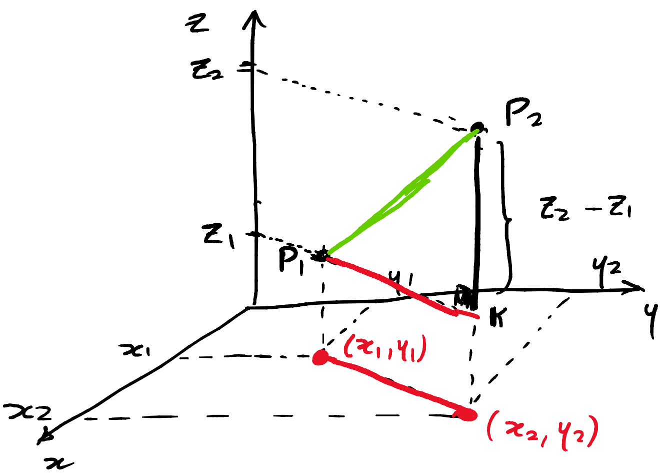
$$\sqrt{(x - x_0)^2} < \sigma$$

1 dimension



$$\sqrt{(x-x_0)^2+(y-y_0)^2} < \delta \qquad 2 \text{ dimensions}$$





$$d(P_1, P_2) = ?$$

$d(P_1, K)$

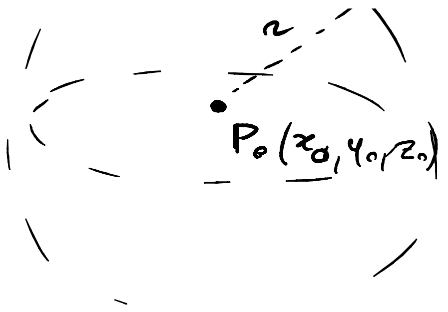
$$\overline{P_1K} = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

$$d(P_1, P_2) = \sqrt{\overline{P_1K}^2 + \overline{P_2K}^2} =$$

$$= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_2-z_1)^2} =$$

$$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$d(P, P_0) =$

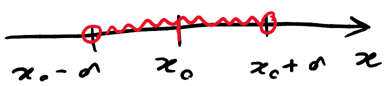


$$d(P, P_0) = r$$

$$\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} = r$$

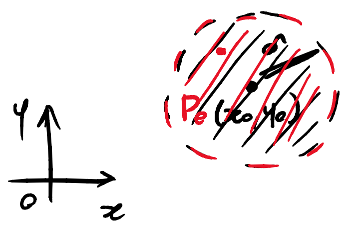
$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$$

$$|x - x_0| = \sqrt{(x-x_0)^2}$$

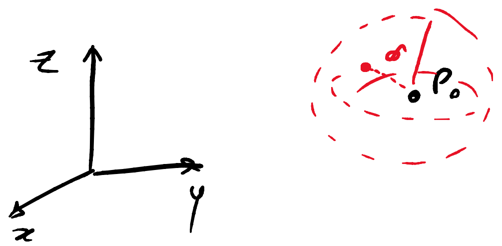


$$\sqrt{(x-x_0)^2} < \sigma$$ 1 dimension

$$d(x, x_0) < \sigma$$



$$\sqrt{(x-x_0)^2 + (y-y_0)^2} < \sigma$$ 2 dimensions



$$\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} < \sigma$$

3 dimensions

In n dimensions, a neighborhood of $P_0(x_1^0, x_2^0, \dots, x_n^0)$ is

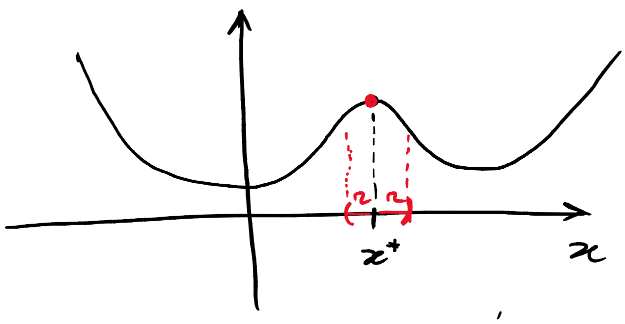
$$\sqrt{(x_1 - x_1^0)^2 + (x_2 - x_2^0)^2 + \dots + (x_n - x_n^0)^2} < \sigma$$

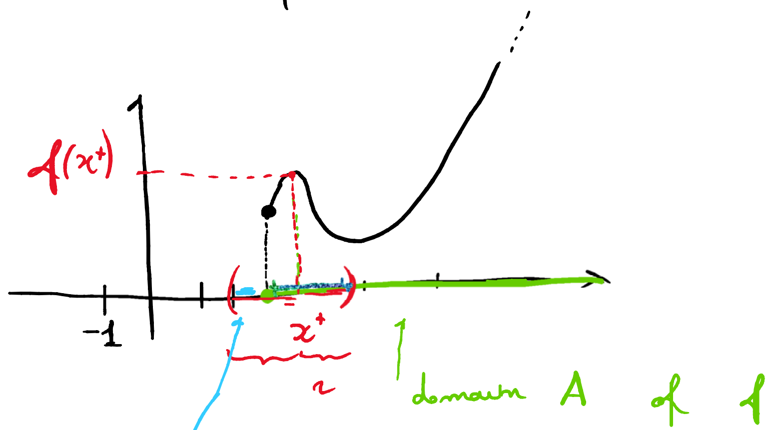
In general, a neighborhood of $\underline{x_0}$ is denoted by $B(\underline{x_0}, \underset{\sigma}{r}) = \{\underline{x} \in \mathbb{R}^n : d(\underline{x}, \underline{x_0}) < \underset{\sigma}{r}\}$

$$f : A \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}$$

$\underline{x}^* \in A$ is a global maximum of f
if $\quad f(\underline{x}^*) \geq f(\underline{x}) \quad \forall \ \underline{x} \in A$

$\underline{x}^* \in A$ is a local (or relative) maximum
of f if $f(\underline{x}^*) \geq f(\underline{x})$
$\forall \ \underline{x} \in B(\underline{x}^*, r) \cap A$



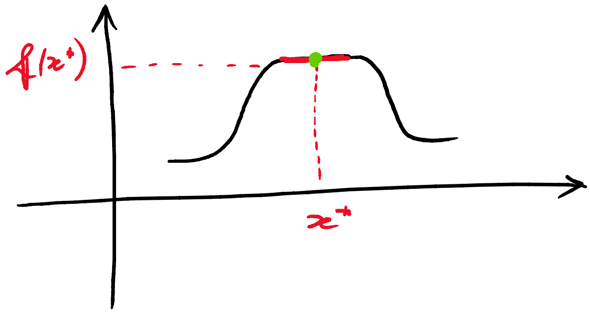


in this case I chose a too-large neighborhood and so the function is not defined for the points in blue

$f(x^*)$ is the greatest for the point in

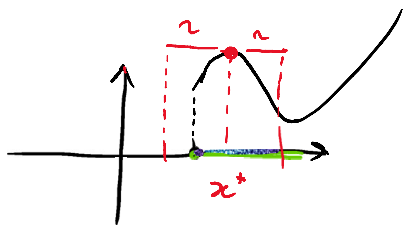
But such an interval is $B(x^*, r) \cap A$.

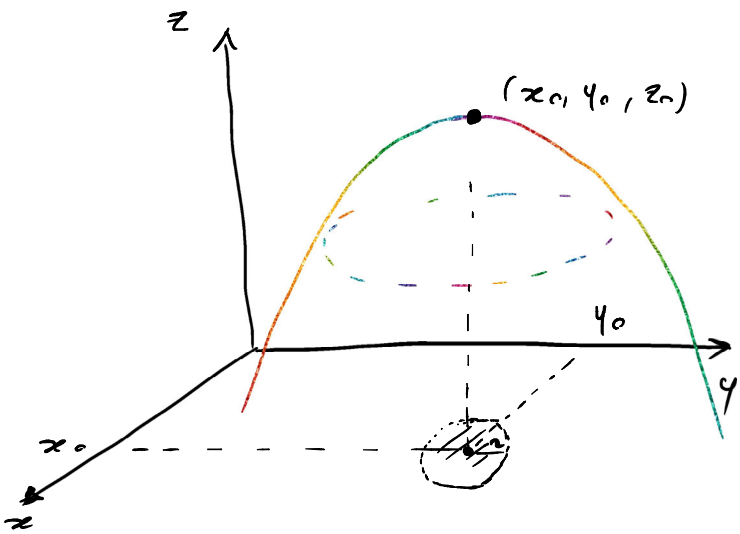
(global)

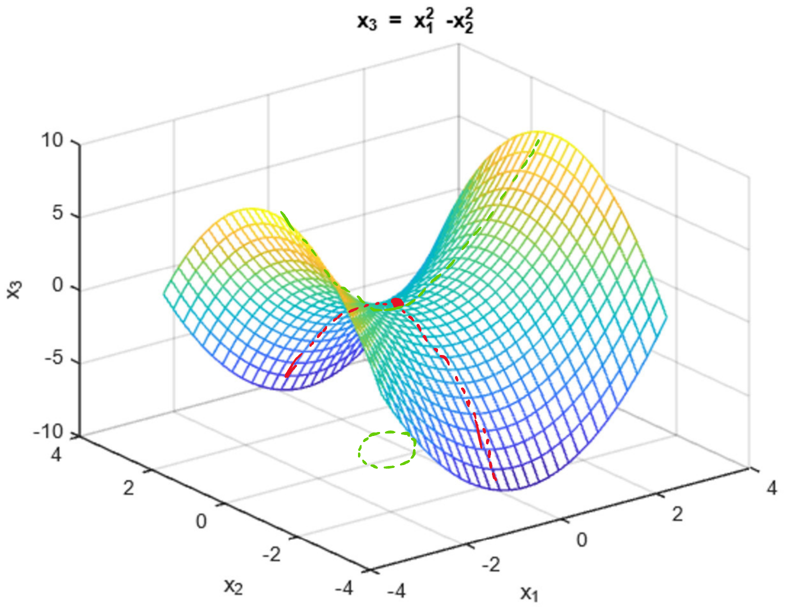


x^* is a (global) maximum but it is not a strict maximum

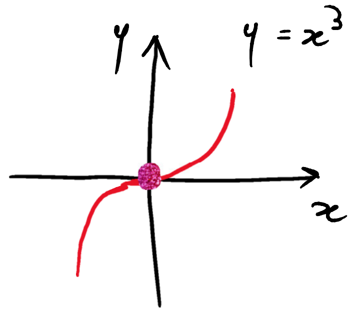
$\underline{x}^* \in A$ is a local (or relative) strict maximum of f if $f(\underline{x}^*) > f(\underline{x})$ $\forall\ \underline{x} \in B(\underline{x}^*, r) \cap A$



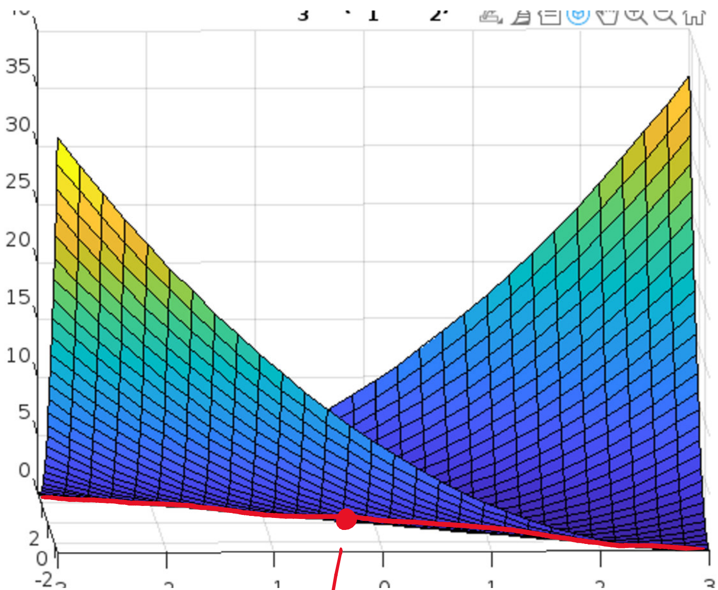




saddle points



$$y' = 3x^2 = 0$$
$$x = 0$$



minima

is a minimum but it is not strict