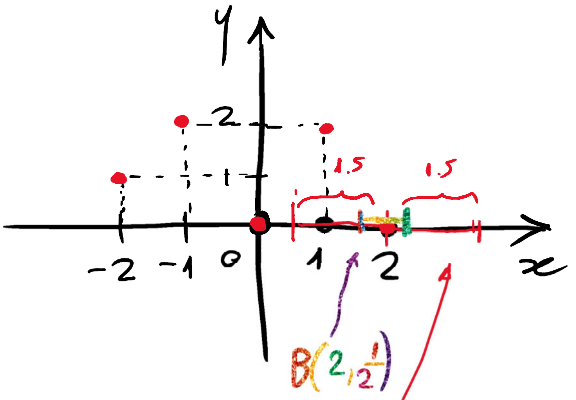
martedì 28 marzo 2023 08:57

$f : A \subseteq \mathbb{R}^n \to \mathbb{R}$

$\underline{x} \in \mathbb{R}^n$ is an accumulation point of $A \subseteq \mathbb{R}^n$ if for all $B(\underline{x}, r)$ there exists a point of A different from $\underline{x}$.

If $n = 1$: example:



$A = \{-2, -1, 0, 1, 2\}$

$f(-2) = 1$

$f(-1) = 2$

$f(0) = 0$

$f(1) = 2$

$f(2) = 0$

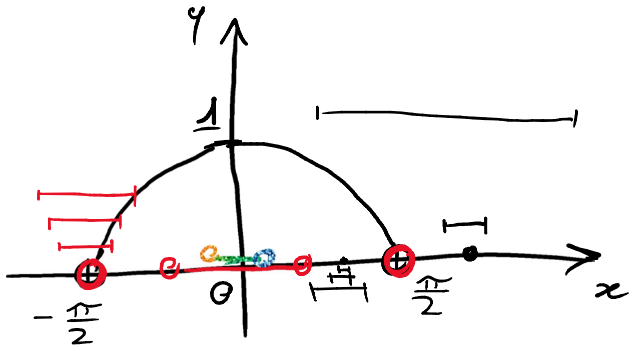
$B(\overset{x}{2}, \overset{r}{\tfrac{3}{2}})$ within this "ball" there is a point different from 2.

$B(2, \frac{1}{2})$ within this ball there is not a point different from 2, so 2 is not an accumulation point of the set A.

Another example with $n = 1$

$A = (-\frac{\pi}{2}, \frac{\pi}{2}) \leftarrow$

$f(x) = \cos(x)$



Let's consider, for example, $x = 0$

$B(0, 1)$

$B(0, \frac{1}{2})$

Independently on the value of r, there are always points different from 0 within the ball.

So 0 is an accumulation point of A.

Also $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ are accumulation points of A even though they are not points of A

Roughly speaking, the accumulation points of a domain A are those points for which I can compute the limit of a function.

$n = 2$.

$$f(x, y) = \ln(4 - x^2 - y^2)$$

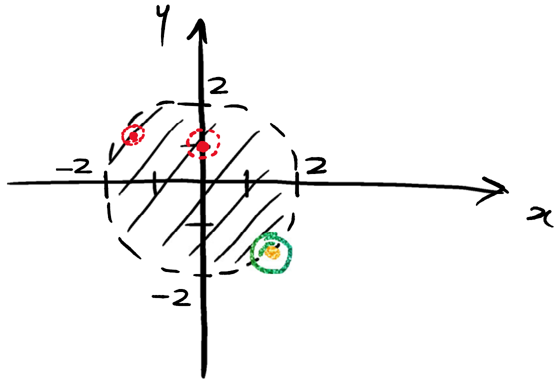
$$4 - x^2 - y^2 > 0$$

$$4 > x^2 + y^2$$

$$x^2 + y^2 < 4$$

$$x^2 + y^2 < 2^2$$

$x^2 + y^2 < 2^2$



In this example, the accumulation points are all the points in the domain plus all the points on the border.

LIMIT OF A FUNCTION

Let $f : A \subseteq \mathbb{R}^n \to \mathbb{R}$ and let $\underline{x}^*$ be an accumulation point of A. Then

$$\lim_{\underline{x} \to \underline{x}^*} f(\underline{x}) = l \quad \text{if}$$

$$\forall \; \varepsilon > 0 \quad \exists \; \delta > 0 :$$

$$|f(\underline{x}) - l| < \varepsilon$$

$$\forall \; \underline{x} \in B(\underline{x}^*, \delta) \cap A - \{\underline{x}^*\}$$

$n = 1$

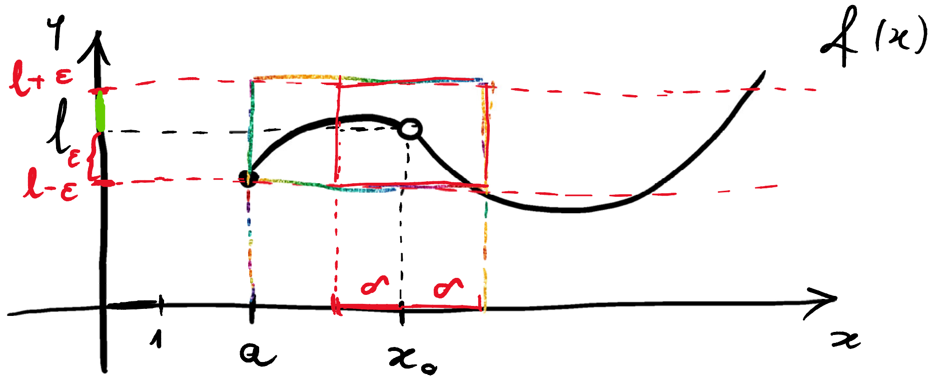
$|A| < r$ is equivalent to $-r < A < r$

$$|f(x) - l| < \varepsilon$$

$$-\varepsilon < f(x) - l < \varepsilon$$

$$l - \varepsilon < f(x) < l + \varepsilon$$

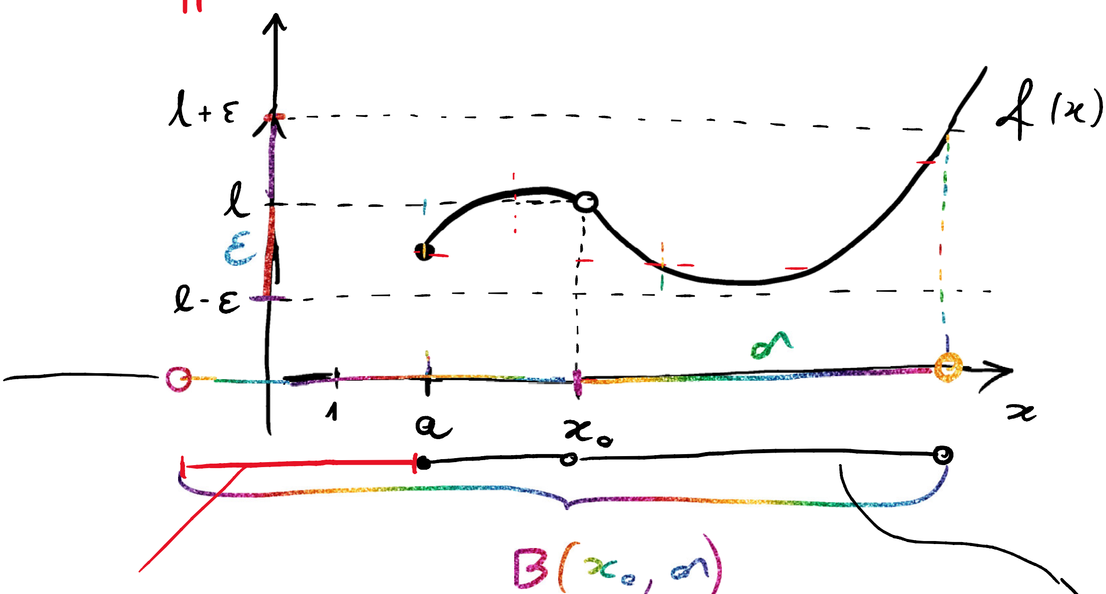
$f(x)$ is trapped between $l - \varepsilon$ and $l + \varepsilon$



$$\mathrm{dom}\, f = [a, x_0) \cup (x_0, +\infty) =$$

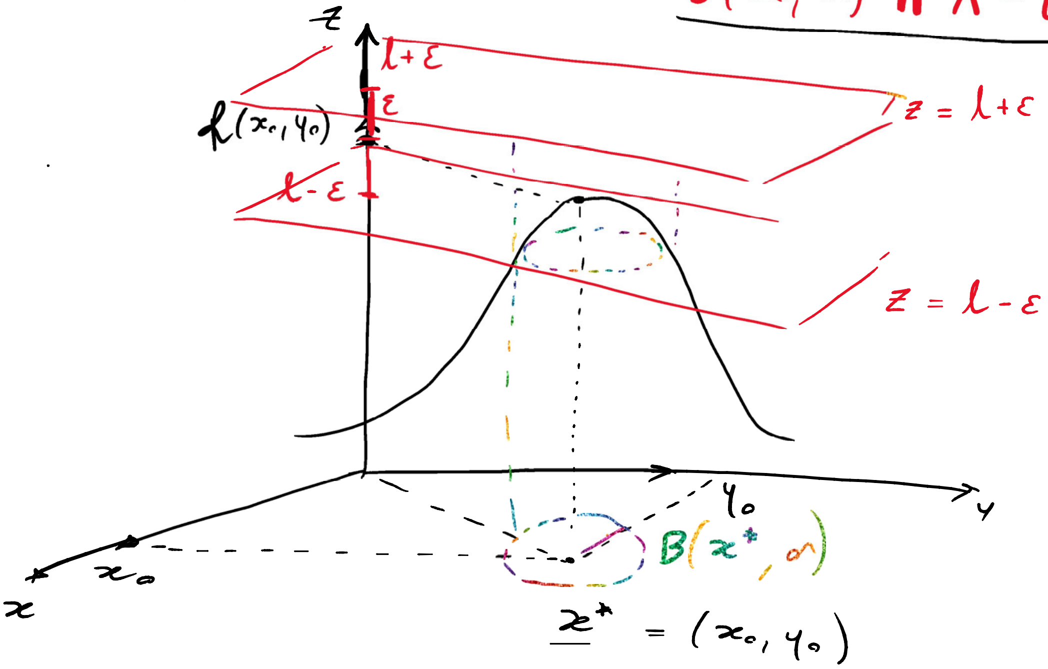
$$= [a, x_0[\cup]x_0, +\infty[$$

I fixed the value of ε ! and I could find a ball $B(x_0, \sigma)$ such that the function is trapped within $l - \varepsilon$ and $l + \varepsilon$.



Here, the function is not defined. That's why in

Here, the function is not defined. That's why in the definition we have $B(x_0, r) \cap A - \{x_0\}$

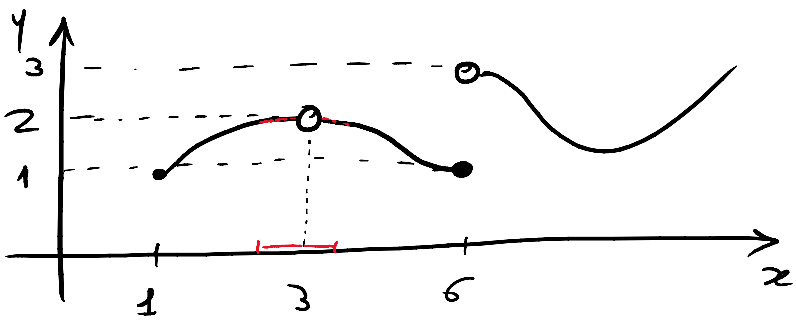


$f : A \subseteq \mathbb{R}^n \to \mathbb{R}$,

f is continuous in $\underline{x}^*$ if

$$\lim_{\underline{x} \to \underline{x}^*} f(\underline{x}) = f(\underline{x}^*)$$

$n = 1$



$$\lim_{x \to 3} f(x) = 2$$

$f(3)$ is not defined

so it is not true that $\lim f(x) = f(3)$

so it is not true that $\lim_{x \to 3} f(x) = f(3)$

So there is a discontinuity

$\lim_{x \to 6^-} f(x) = 1$ $\qquad$ $\lim_{x \to 6^+} f(x) = 3$

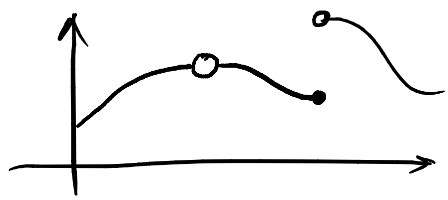
I am approaching 6 from the left

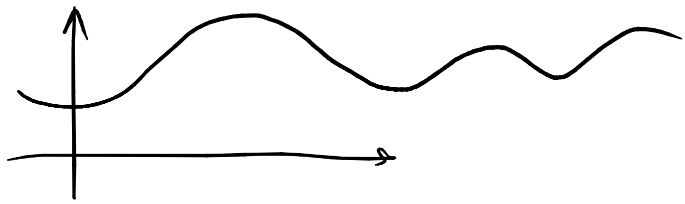
$\lim_{x \to 6} f(x)$ does not exist.

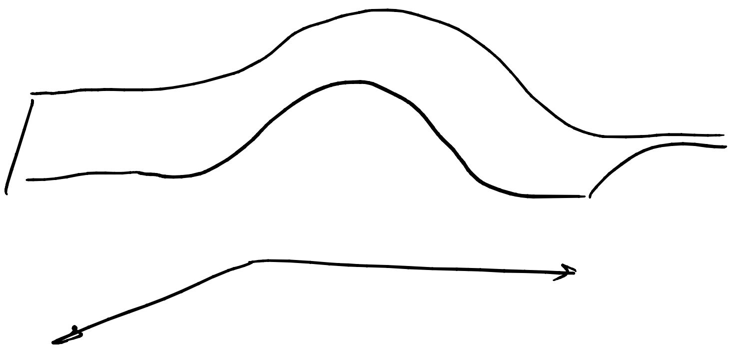
so again it is not true that $\lim_{x \to 6} f(x) = f(x)$

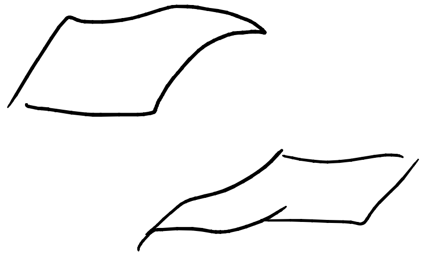
This is another discontinuity

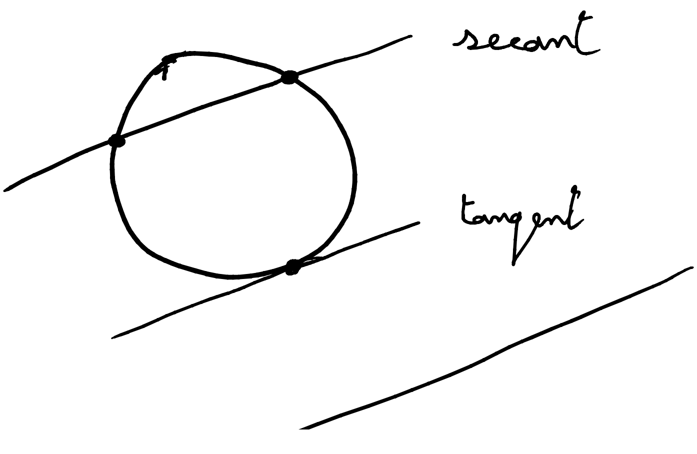
Roughly speaking a continuous function is a function that I can draw without leaving the pen from the paper.

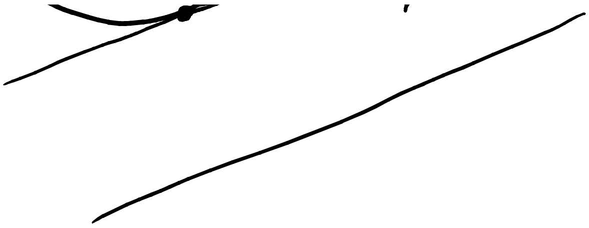


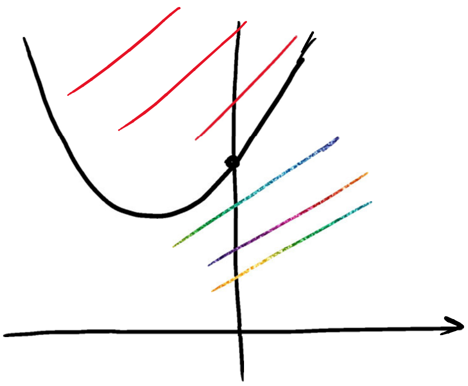


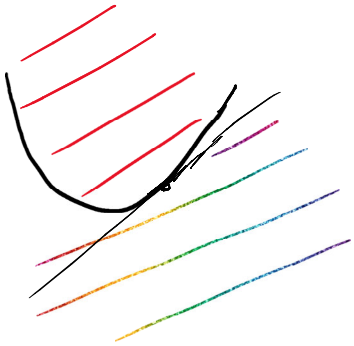


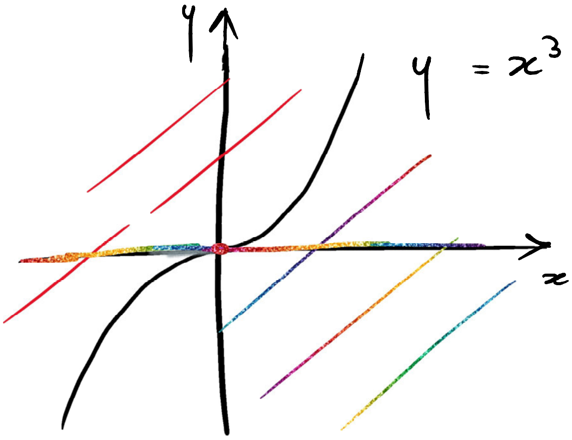


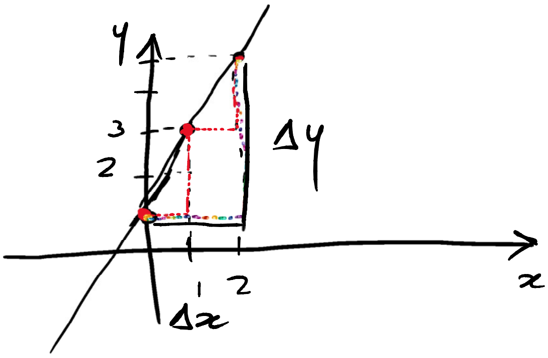












slope intercept

$$y = mx + q$$

b

~

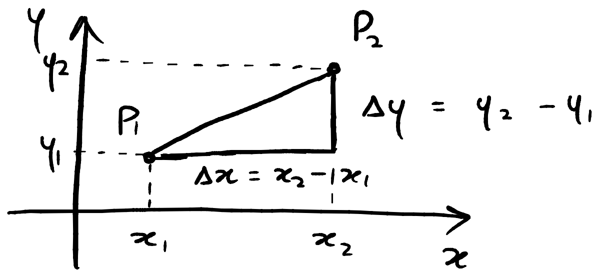
$$y = 2x + 1$$

If I move to the right of Δx units I go up of $2 \cdot \Delta x$ units

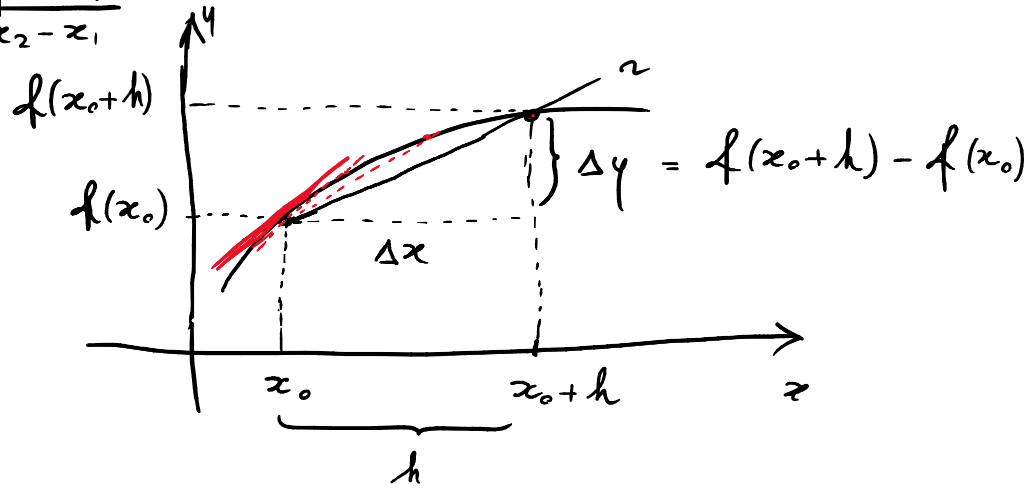
$$\Delta y = 2 \Delta x$$

In general $\Delta y = m \Delta x$

$$m = \frac{\Delta y}{\Delta x}$$



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



The slope of the line r is:

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x_0+h) - f(x_0)}{h}$$

when $h \to 0$ the line becomes tangent to the function

$$m = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} = f'(x_0) = \frac{df}{dx}(x_0) = Df(x_0)$$

The derivative of the function f in x_0 is the slope of the tangent line to the function in x_0.

$$D\, x^{\alpha} = \alpha\, x^{\alpha - 1}$$

$$D(\sin(x)) = \cos x$$

$$D(\cos x) = -\sin x$$

$$D\big(f(x)\, g(x)\big) = f'(x)\, g(x) + f(x)\, g'(x)$$

$$h(x) = x^2 \operatorname{sen} x$$

$$h'(x) = 2x \cdot \sin x + x^2 \cdot \cos x$$

$$D\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)\, g(x) - f(x)\, g'(x)}{g^2(x)}$$

$$D\left[f(g(x))\right] = f'(g(x))\, g'(x)$$

$$D\, e^x = e^x$$

$$D \ln|x| = \frac{1}{x}$$

$$e^{x^2 - \sin x}$$

$$f(x) = e^x$$

$$g(x) = x^2 - \sin x$$

$$e^{x^2 - \sin x} \quad D\left(x^2 - \operatorname{sen} x\right) =$$

$$= e^{x^2 - \sin x}\,(2x - \cos x)$$