Find the time required for an initial amount of \$ 5000 to be equal to a final amount of \$ 9000 deposited at 4% compounded quarterly.

$$i_4 = 4\%$$

$$W = W_0 (1 + i_1)^{t_1} = W_0 (1 + i_1)^{t \cdot 1}$$

$$W = W_0 (1 + i_4)^{t_4} = W_0 (1 + i_4)^{t \cdot 4}$$

$$\cancel{W_0} (1 + i)^t = \cancel{W_0} (1 + i_4)^{4t}$$

$$1 + i = (1 + i_4)^4$$

$$i = (1 + i_4)^4 - 1 = 1{,}04^4 - 1 \cong 0.1699$$

$$W = W_0 (1 + i)^t$$

$$\frac{W}{W_0} = (1 + i)^t$$

$$\ln \frac{W}{W_0} = t \cdot \ln (1 + i)$$

$$\ln \frac{W}{W_0} = t \cdot \ln \left(\cancel{1} + (1 + i_4)^4 - \cancel{1}\right)$$

$$\ln \frac{W}{W_0} = t \ \ln (1 + i_4)^4$$

$$\ln \frac{W}{W_0} = 4t \ln(1 + i_4)$$

$$t = \frac{\ln \frac{W}{W_0}}{4 \ln(1 + i_4)} =$$

$$= \frac{\ln \frac{9}{5}}{4 \ln 1.04} \text{ years} \cong 3{,}7 \text{ years}$$

OR

$$W = W_0 (1 + i_4)^t$$

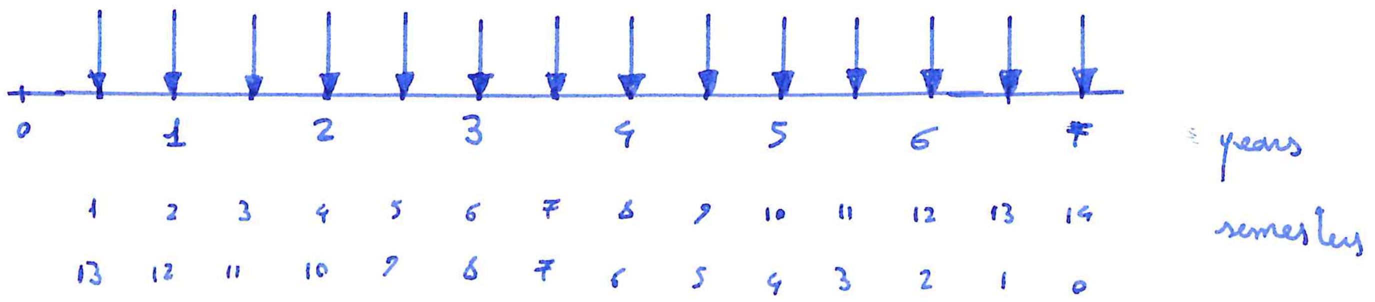
$$\frac{W}{W_0} = (1 + i_4)^t$$

$$\ln \frac{W}{W_0} = t \ln(1 + i_4)$$

$$t = \frac{\ln \frac{W}{W_0}}{\ln(1 + i_4)} = \frac{\ln \frac{2}{5}}{\ln 1.04} \text{ quarters} \cong$$

$$\cong 14{,}98 \text{ quarters} = \frac{14.98}{4} \text{ years} \cong 3.7 \text{ years}$$

Find the future value of an ordinary annuity with $R = \$ 9200$, 10% interest compounded semiannually for 7 years, where each payment takes place at the end of each semester.



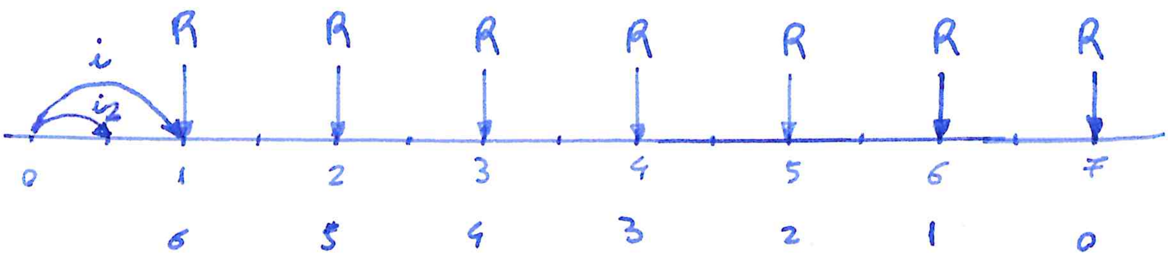
$$W = R + R(1+i_2)^1 + R(1+i_2)^2 + \cdots + R(1+i_2)^{13} =$$

$$= R\left[1 + (1+i_2)^1 + (1+i_2)^2 + \cdots + (1+i_2)^{13}\right] =$$

$$= R \, \frac{1 - (1+i_2)^{14}}{1 - (1+i_2)} = R \, \frac{(1+i_2)^{14} - 1}{i_2} =$$

$$= \frac{1.1^{14} - 1}{0{,}1} \; 9200 \,\$ \cong 257369.85 \,\$$$

Repeat the exercise in the case where the payments occur once a year.



$$W = R + R(1+i)^1 + \dots + R(1+i)^6 =$$

$$= R \, \frac{(1+i)^7 - 1}{i}$$

$$\cancel{W_0}(1+i)^t = \cancel{W_0}(1+i_2)^{t_2}$$

$$(1+i)^{\cancel{t}} = (1+i_2)^{\cancel{t}\cdot 2}$$

$$i = (1+i_2)^2 - 1$$

$$W = R \, \frac{(1+i)^7 - 1}{i} =$$

$$= R \, \frac{(1+i_2)^{14} - 1}{(1+i_2)^2 - 1} =$$

$$= 9200\,\$ \, \frac{1.1^{14} - 1}{1.1^2 - 1} \cong 122557.07\,\$$$

Given the matrix

$$\underline{\underline{A}} = \begin{bmatrix} 10 & -8 & 0 \\ 0 & 6 & 0 \\ 0 & -4 & 10 \end{bmatrix},$$

compute its eigenvalues.
For each eigenvalue, compute an associated eigenvector.

$$\det(\underline{\underline{A}} - \lambda \underline{\underline{I}}) = 0$$

$$\begin{vmatrix} 10-\lambda & -8 & 0 \\ 0 & 6-\lambda & 0 \\ 0 & -4 & 10-\lambda \end{vmatrix} = 0$$

$$(10-\lambda) \begin{vmatrix} 6-\lambda & 0 \\ -4 & 10-\lambda \end{vmatrix} = 0$$

$$(10-\lambda)(6-\lambda)(10-\lambda) = 0$$

$$\lambda = 10 \quad \vee \quad \lambda = 6$$

$$(\underline{\underline{A}} - 10 \underline{\underline{I}}) \underline{v} = \underline{0} \qquad \underline{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 0 & -8 & 0 \\ 0 & -4 & 0 \\ 0 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -8y = 0 \\ -4y = 0 \\ -4y = 0 \end{cases} \qquad y = 0$$

$$\underline{v} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$(\underline{\underline{A}} - 6\underline{\underline{I}})\,\underline{v} = \underline{0}$$

$$\begin{bmatrix} 4 & -8 & 0 \\ 0 & 0 & 0 \\ 0 & -4 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} 4x - 8y = 0 \\ -4y + 4z = 0 \end{cases} \qquad \begin{cases} x - 2y = 0 \\ y = z \end{cases}$$

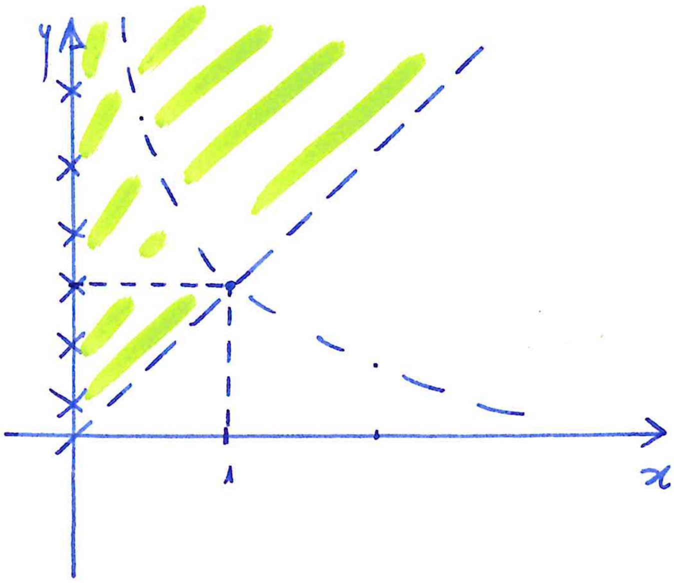
$$\begin{cases} x - 2z = 0 \\ y = z \end{cases} \qquad \begin{cases} x = 2z \\ y = z \end{cases}$$

$$\underline{v} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

Find the domain of the following function:

$$z = \frac{\ln\left(x\sqrt{y-x}\right)}{xy-1}$$

$$\begin{cases} x > 0 \\ y - x > 0 \\ xy - 1 \neq 0 \end{cases} \qquad \begin{cases} x > 0 \\ y > x \\ xy \neq 1 \end{cases}$$



Compute the gradient of the following function:

$$f(x,y) = xy \ln(x+y)$$

$$f'_x = y \ln(x+y) + xy \frac{1}{x+y} =$$

$$= y\left[\ln(x+y) + \frac{x}{x+y}\right]$$

$$f'_y = x\left[\ln(x+y) + \frac{y}{x+y}\right]$$

$$f(x,y) = \tan^{-1}(x^2 - y^2) - \frac{1}{3}x^3$$

$$f'_x = \frac{1}{1+(x^2-y^2)^2} 2x - \frac{1}{3}\cdot 3x^2 =$$

$$= \frac{2x}{1+(x^2-y^2)^2} - x^2$$

$$f'_y = \frac{1}{1+(x^2-y^2)^2}(-2y) = -\frac{2y}{1+(x^2-y^2)^2}$$

$$\begin{cases} \dfrac{2x}{1+(x^2-y^2)^2} - x^2 = 0 \\ -\dfrac{2y}{1+(x^2-y^2)^2} = 0 \end{cases} \qquad \begin{cases} \dfrac{2x}{1+x^4} = x^2 \\ y = 0 \end{cases}$$

$$\frac{2x}{1+x^4} = x^2$$

$$x^2 - \frac{2x}{1+x^4} = 0$$

$$x\left(x - \frac{2}{1+x^4}\right) = 0$$

$$x = 0 \quad \vee \quad x = \frac{2}{1+x^4}$$

$$x(1+x^4) = 2$$

$$x + x^5 = 2$$

$$x^5 + x - 2 = 0 \qquad x = 1$$

	1	0	0	0	1	-2
1		1	1	1	1	2
	1	1	1	1	2	//

$$(x-1)(x^4+x^3+x^2+x+2)=0$$

$$x=1$$

$$A(0,0) \quad \vee \quad B(1,0)$$

$$f'_x = \frac{2x}{1+(x^2-y^2)^2} - x^2$$

$$f''_{xx} = \frac{2[1+(x^2-y^2)^2] - 2x\cdot 2(x^2-y^2)\cdot 2x}{[1+(x^2-y^2)^2]^2} - 2x =$$

$$= \frac{2+2(x^2-y^2)^2 - 8x^2(x^2-y^2)}{[1+(x^2-y^2)^2]^2} - 2x$$

$$f''_{xx}(A) = f''_{xx}(0,0) = 2$$

$$f''_{xx}(B) = \frac{2+2-8}{4} - 2 =$$

$$= -1-2 = -3$$

ou

$$f''_{xx}(0,0) = \lim_{h\to 0} \frac{f'_x(h,0) - \cancel{f'_x(0,0)}}{h} =$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{2h}{1+h^4} - h^2\right] =$$

$$= \lim_{h\to 0} \left(\frac{2}{1+h^4} - h\right) = 2$$

$$\cancel{f''_{xx}(B) = f''_{xx}(1,0)}$$

$$\cancel{= \lim_{h \to 0} \frac{f'_x(1+h,0) - f'_x(1,0)}{h}} =$$

$$\cancel{= \lim_{h \to 0} \frac{1}{h}\left[\frac{2(1+h)}{1+(1+h)^4} - (1+h)^2 \right]}$$

$$f''_{xy} = 2x \frac{\partial}{\partial y}\left[1+(x^2-y^2)^2\right]^{-1} =$$

$$= 2x(-1)\left[1+(x^2-y^2)^2\right]^{-2} 2(x^2-y^2)(-2y) =$$

$$= \frac{8xy(x^2-y^2)}{\left[1+(x^2-y^2)^2\right]^2}$$

$f''_{xy}(A) = 0$ $\qquad$ $f''_{xy}(B) = 0$

$$f'_y = -\frac{2y}{1+(x^2-y^2)^2}$$

$$f''_{yy} = -\frac{2\left[1+(x^2-y^2)^2\right] - 2y \cdot 2(x^2-y^2)(-2y)}{\left[1+(x^2-y^2)\right]^4} =$$

$$= -\frac{2+2(x^2-y^2)^2+8y^2(x^2-y^2)}{\left[1+(x^2-y^2)\right]^4}$$

$f''_{yy}(A) = -2$ $\qquad$ $f''_{yy}(B) = -\frac{2+2}{16} = -\frac{1}{4}$

$$\underline{\underline{H}}f(A) = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \qquad A: \text{ saddle point}$$

$$\underline{\underline{H}}f(B) = \begin{bmatrix} -3 & 0 \\ 0 & -\frac{1}{4} \end{bmatrix} \qquad B: \text{ local maximum}$$