

$$f(x, y) = x^2 - y^2$$

$$k = x^2 - y^2$$

$$y^2 = x^2 - k$$

$$y = \pm \sqrt{x^2 - k}$$

$$f(x, y) = (x + y^2) \sin(x - y) \quad \underline{x}_0 = (0, 0)$$

$$f(x, y) = f(x_0, y_0) + \nabla f(\underline{x}_0)^T (\underline{x} - \underline{x}_0) + \frac{1}{2} (\underline{x} - \underline{x}_0)^T \underline{H}f(\underline{x}_0) (\underline{x} - \underline{x}_0)$$

$$f(\underline{x}_0) = f(x_0, y_0) = f(0, 0) = 0$$

$$f'_x = \sin(x - y) + (x + y^2) \cos(x - y)$$

$$f'_x(\underline{x}_0) = f'_x(x_0, y_0) = f'_x(0, 0) = 0$$

$$f'_y = 2y \sin(x - y) + (x + y^2) \cos(x - y) (-1)$$

$$f'_y(\underline{x}_0) = f'_y(x_0, y_0) = f'_y(0, 0) = 0$$

$$f''_{xx} = \cos(x - y) + \cos(x - y) + (x + y^2) (-\sin(x - y))$$

$$f''_{xx}(0, 0) = 2$$

$$f''_{xy} = -\cos(x - y) + 2y \cos(x - y) + (x + y^2) \sin(x - y)$$

$$f''_{yx} = 2y \cos(x - y) - \cos(x - y) + (x + y^2) \sin(x - y)$$

$$f''_{xy}(0, 0) = -1$$

$$f''_{yy} = 2 \sin(x - y) + 2y \cos(x - y) (-1) - 2y \cos(x - y) + (x + y^2) \sin(x - y)$$

$$f''_{yy}(0, 0) = 0$$

$$H_f(0,0) = \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix}$$

$$T_2(x,y) = \frac{1}{2} \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} =$$

$$= \frac{1}{2} \begin{bmatrix} 2x - y & -x \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} =$$

$$= \frac{1}{2} (2x^2 - xy - xy) = \frac{1}{2} (2x^2 - 2xy) = x^2 - xy$$

