International Finance and Economics
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Mathematical methods for economics and finance

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MOD B - PART 2 - MatLab

MatLab= MATrix LABoratory (environment for the scientific calculation and numerical simulations)

MatLab Desktop :

- **Command Window:** (to insert commands and instructions, >> is the prompt)
- **Command History:** chronological sequence of the executed instructions
- **Workspace:** operative memory containing the variables, array
- **Current Directory:** containing all the files and folders

VECTORS

A **vector** composed by n elements is given by n ordered real numbers

Row vector Ex: $\underline{x}$=(1, 3, -4, 11) composed by 4 elements (dimension 4)

Column vector Ex:

$$\underline{y}=\begin{bmatrix}1\\-1\\0\end{bmatrix}$$ composed by 3 elements (dimension 3)

A real number z=(133) is a row or column vector having one element

Save a real number

To assign a name to a number you must use the symbol **=**

Ex: define variable a with value 25

```
>> a=25
```

Such variable will be saved in the Workspace!

Notice: MatLab is *case sensitive*

Notice: ; after the command: the result of the instruction will not appear in the command window

Ex 1

Save in MatLab the following real numbers:

$$A = \frac{12}{5}; B = 3^6; C = 2,1 \cdot 7$$

Notice: 2,1 must be written as 2.1 (the point separates decimals)

And calculate:

$$a = A + \frac{B}{C}; b = \frac{C - a}{B}; c = bB^A$$

Some initial commands

>>clc cleans command window

>>clear cleans workspace

>> ↑ recall the last executed command

>>doc "argument" consults MatLab help on the «argument»

>>format long activates 14 decimals format

>>format short activates 4 default decimal format

Use elementary functions

Ex: to save number e^2 …we need to know the syntax of the required function!

Notice: >>doc elfun to consult MatLab help on elementary functions. Thus obtaining

```
>> w=exp(2)
```

Ex 2

Save in MatLab the following real numbers:

$$x = \ln(4); y = \frac{1}{8}; z = -6$$

And calculate:

$$p = e^{x} - \ln 2; q = \sqrt{y} z^{3}; r = |-10 + 2z|$$

Save a vector

ROW VECTOR: list the elements separated by a **space** or a **comma ,** inside square brackets

Ex: save the row vector (10,-1,5)

```
>> A=[10 -1 5]
```

COLUMN VECTOR: list the elements separated by a **semicolon ;** inside square brackets, or write a row vector and put **apostrophe '** at the end

Ex: save the column vector with elements 3, 7, 9

```
>> B=[3;7;9]
```

Work with vectors

Notice: each element of a vector is identified by an **index** (which defines its position)

Hence: to **access to an element** of the vector the index must be specified between rounded bracket

Ex: to select the second element of vector B

```
>> b=B(2)
```

Notice: in such a way you can modify the value of an element of a vector

Ex: change the value of the first element of vector A into 7

```
>> A(1)=7
```

Work with vectors

Notice: it is also possible to **delete an element form a vector**
By substituting the selected element with an empty vector **[]**

Es: delete the third element from vector MM

```
>> MM(3)=[]
```

Ex 3

Save the following vectors:

$$x = \left(-2, \sqrt{3}, e, 10\right); y = \begin{pmatrix} 0 \\ -1 \end{pmatrix}; z = \begin{pmatrix} 3/5 \\ 7 \\ -4 \end{pmatrix}$$

Which is their dimension?

Transform the vector x in a column vector

Change the second element of vector y into 5

Delete the first element of z

Operator :

An **equally spaced vector** from i to j is a vector in which

i is the first element

j is the last element

The distance between each element and the previous one is constant and given by **p**

The command i:p:j is used to create an equally spaced vector from element i to element j with step p

Notice: if i>j a negative p must be used

Notice: if p is not specified then p=1 will be assumed

Operator :

Ex: save a row vector Y with equally spaced elements from 0 to 40 and step 4

```
>> Y=0:4:40;
```

Ex: from vector Y, create a new vector given by the elements of Y from the second to the sixth

```
>> Y1=Y(2:6)
```

Notice: in this last case p is not specified since the required step is equal to 1

Ex 4

Save the equally spaced column vector P having elements from -10 to 60 and step 2.

Save the equally spaced row vector Q having 11 elements from -1 to -6. Which step do you have to use?

Trasform Q into a column vector and then delete its elements from the 2nd to the 5th

Operator linspace

To create an equally speced vector from i to j composed by n elements use the command **linspace(i,j,n)**

Ex: save the row vector V having 20 equally spaced elements from -2 to 6

```
>> V=linspace(-2,6,20)
```

Ex: delete the elements of V having even indexes

```
>> V(2:2:20)=[]
```

Ex 5

Save the row equally spaced vector A with 60 elements from 3 to 20 and the equally spaced column vector B with 10 elements from 2 to -5

Which step has been used in the two cases?

1) Operation: sum

Consider two vectors X and Y of the same type (both row or column) and the same dimension (same number of elements)

Then it is possible the **sum (difference)** X+(-)Y between the two vectors, thus obtaining Z of the same type and dimension of the initial vectors. Each element of Z is given by the sum (difference) of the elements of X and Y having the same index

You have to use the operator **+ (-)**

Ex: P row vector with equally spaced elements from 6 to 20 with step 2 and Q row vector with 8 equally spaced elements form 1 to 3. Compute R=P-Q

```
>> P=6:2:20;
>> Q=linspace(1,3,8);
>> R=P-Q
```

Notice: if it is computed X+1, the number 1 is added to each element of X

Ex 6

Equally spaced vectors: save row A with 16 elements from 3 to 20 and column B with elements from 0 to 90 step 5

1. Can you sum the two vectors?
2. Trasform the two vector in order to make their sum possible

2) Operation: product between a scalar number and a vector

Consider a vector X and a real number k.

The **scalar moltiplication** kX gives a vector Z (same type and dimension of X). Each element of Z is given by the product of number k with the correspondent element of X

The command to be used is *

Ex: compute S=-10R

```
>> S=-10*R
```

Ex 7

Save the row vector X with elements -1,0,5,7 and the equally spaced row vector Y with 4 elements form 10 to -8.

1. Compute Z=X+Y.
2. Compute V=0.5X.
3. Compute Z-2V.

3) Operation: punctual product

It is used to compute the products between two vectors, element by element.

Consider two vectors X and Y of the same type and same dimension

It is possible to compute the **punctual product X.*Y** between the two vectors. The vector Z, of the same time and dimension of X, is obtained and each element of Z is given by the product of the elements of X and Y having the same position

The command to be used is **.***

Ex: consider the two column vectors A with elements 0,-1,3 and B with elements -3,2,2. Compute their punctual product.

```
>> A=[0;-1;3];B=[-3 2 2]';
>> C=A.*B
```

4) Operation: punctual division

Consider two vectors X and Y of the same type and dimension

Then it is possible the **punctual division X./Y** thus obtaining Z: each element is given by the division between the corresponding element of X and Y.

The command to be used is **./**

Notice: if one lement of Y is zero, the vector Z is computed but Inf or NaN will be notified

Es: consider two row vectors A=(0,-1,1) and B=(-1,-2,-3). Compute the punctual division A./B

```
>> A=[0 -1 1];B=[-1 -2 -3];
>> C=A./B
```

Ex 8

Save the row vector X with elements -1,0,1,2,3 and the row vector Y with equally spaced elements from -3 to 1 with step 1.

1. Calculate Z as the puntual product between X and Y. Does the commutative property hold?

2. Obtain V by dividing X with respect to Y. Is it possible on set R of real numbers? And with MatLab?

3. Substitute the null elements of Y with the unitary value and calculate V=X./Y+3X-1.

5) Operation: punctual power

Consider a vector X and a real number k. Then it can be computed the **punctual power X.^k** thus obtaining a vector in which each element is obtained as the power-k of the correspondent element of X.

The command is **.^**

Notice: if for some elements of X the power-k cannot be computed, it will be notified. The other elements will be calculated. If the operation is defined only in complex set, it will be computed.

Ex: row equally spaced vector A with 10 elements from 7 to 21 and k=1/3. Compute A.^k

```
>> A=linspace(7,21,10);
>> B=A.^(1/3)
```

5.1) Operation: punctual power

Consider two vectors X and Y of the same type and dimension. Then it can be computed the **punctual power X.^Y** thus obtaining a vector in which each element is obtained by raising each element of X to the correspondent element of Y.

The command is, again, **.^**

Ex: A is the row with elements -2,0,4 while B is the row with elements 0.5,-1,2.

```
>> A=[-2 0 4];
B=[0.5 -1 2];A.^B

ans =

   0.0000 + 1.4142i        Inf + 0.0000i   16.0000 + 0.0000i
```

Notice: the first element of the obtained vector is a complex number while the second element cannot be computed (Inf is notified).

Ex 9

Save the column vector X with elements -10,0,1,2,3 and the column vector Y with 6 equally spaced elements from -1 to 1.5.

1. Calculate Z by elevating each element of X to the power -1. Is it possible in R? With MatLab?
2. Calculate W=X.^Y and observe the result to understand its meaning.

6) Operation: a function applied to a vector

Consider a vector X (row or columnwith dimension n) and let **f be a function of one real variable**.

Then it is possible to calculate **f(X)** thus obtaining a vector Z (row or column with dimension n) such that each element of Z is given by the application of function f to the correspondent element of vector X.

Ex: A=[2,3,7,9] and f(x)= e^{x-4}. Calculate f(A)

```
>> A=[2,3,7,9];
>> B=exp(A-4);
```

Ex 10

Save the following row vectors: X with equally spaced elements from 10 to 30 and step ¼ and Y with elements -1,3,7,0

1. Let $f(x)=\ln(x)-(x+1)^{2/3}$. Calculate Z=f(X) and observe the result to understand its meaning.
2. Let $f(x)=1/x+2x-\ln(2x)$. Calculate W=f(Y) and observe the result to understand its meaning.

Homeworks

1.1 Save the row vector X with 24 equally speced elements form -5 to 7

A. Calculate $Y = \sqrt{2}X$

B. From Y delete the elements having indexes that are multiple of 3

C. Create Z with equally spaced elements form 1 to 16 step 1

D. Calculate, if possible, $V = Y + Z$

E. Let $f(x) = \dfrac{\sqrt[3]{2x^2 - 3}}{0.2x}$, calculate $W = f(Z)$

Notice: use the punctual operators when it is necessary!

Homeworks

1.2 Save the column vector A with equally speced elements form 9 to -18 step-0,5 and the row vector B with elements (7,-1,3,5,8,e).

A. Which is the dimension of A?

B. Substitute the 10th element of A with -3

C. Transform the row vector B into the column vector $B1$

D. Let $f(x) = e^{\frac{|x+2|}{x^2}}$, calculate $C = f(\mathrm{B1})$

E. Let $f(x) = \sqrt[3]{x} - 3$, calculate $D = f(A)$

Homeworks

1.3 Create the row vector X with 8 equally speced elements from 1 to 25, the row vector Y with equally speced elements from 10 to 90 step 10, the column vector Z with 8 elements all equal to 1

A. It it possible to compute $X+Y$? Why?

B. Delete from Y the 6th element

C. Convert Z into a row vector V

D. Calculate $W=(X./Y).^\wedge V$

E. Let $f(x)=(x+30)\ln(x+30)$, calculate $f(W)$

Homeworks

1.4 Save the row vectors x=(-1,1,-2,2,-3,3), y=(7,5,-1,3,2,0) and $z=(e,e^2,e^3,e^4,e^5,e^6)$

Notice: find a way to define z without listing all its elements

A. Save $a = e^3, b = \log_2 10, c = \sqrt[3]{5^2}$

B. Calculate $V = (ax - by)cz$

C. Let $f(x) = 10x^{0.5}$, calculate $f(V)$.

Save a Matrix

Consider a Matrix having m rows and n columns

- To define a matrix the elements must be written **row by row**, between square brakets

- When changing the row use the **semicolon**

The obtained matrix is of mxn kind, that is its dimension is mxn

EX. Save the following Matrix: $A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$

```
>> A=[1 3 5; 2 4 6]
```

To obtain the dimension of a matrix A the command is: **size(A)**

To obtain the **transpose matrix** (by changing rows with columns) the following command must be given:

```
>> A'
```

Work with matrices

Notice: A matrix can be saved by using row vectors or column vectors previously defined: it can be considered as a **column vector having elements given by row vectors or as a row vector in which each element is a column vector**

Ex. For the previously given matrix A:

```
>> u=1:2:5;
>> v=2:2:6;
>> A=[u;v]
```

Notice: each element of a matrix is identified by two indexes **the row index and the column index**

For instance in the previous examples the element 6 belongs to the second row and third column (it has indexes 2,3)

Hence: to **select an element of a matrix** the name of the matrix must be followed by the indexes between round brackets:

```
>> A(2,3)
```

Work with matrices

Notice: by considering the indexes of an element it is possible to **change the value of one element of the matrix**

```
>> A=[1 3 5; 2 4 6]
```

Ex. Substitute the element having value 3 with the new value -1

```
>> A(1,2)=-1

A =

     1    -1     5
     2     4     6
```

Work with matrices

(1) To **select a row vector or a column vector** of the given matrix it must be used the operator **:**

A(:,j) gives column j of matrix A,   A(i,:) gives row i of matrix A

Ex.

- select the first column of A

```
>> A(:,1)
```

- select the second row of A

```
>> A(2,:)
```

Work with matrices

(2) To **select a sub matrix** it must be specified the interval of the row-indexes or column-indexes

EX.

Select the matrix from A composed by all the row of matrix A and the first two columns of matrix A

```
>> A(:,1:2)
```

Substitute the second row of A with an equally spaced vector given by elements from 2 to 6 with step 2

```
>> A(2,:)=2:2:6
```

Select the matrix from A composed by all the row of matrix A and the first and third column of A

```
>> A(:,[1,3])
```

Work with matrices

SUMMARIZING

A(i,j)	select the element **(i,j)**
A(i,:)	select **i-row**
A(:,j)	select **j-column**
A(:,[n,m])	select **n-column and m-column**

Work with matrices

(3) It is possible to **delete a row or a column** (thus changing the dimension of the matrix)

Ex.

Consider matrix

```
>> A=[1 2 3; 4 5 6; 7 8 9]

A =

     1     2     3
     4     5     6
     7     8     9
```

Delete the first row

```
>> A(1,:)=[]
```

EX 11

- Save the following matrix by indicating its elements:

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$$

- Save the following vectors v1=(1,2,3), v2=(3,4,5), v3=(-1,0,-1). Define matrix B whose rows are given by v1, v2 and v3.

-Select form A the submatrix C: the rows are given by the first two rows of A while the columns are given by the last two columns of A

- Substitute the first row of B with vector [1,1,1], and the null element of B with the value 5

- Create form B the matrix D by deleting the second row. Which is the dimension of D?

- Obtain matrix E by transposing D

1) OPERATION: SUM

Let A and B be two matrices with same dimension (same number of row and columns). Then it is possible the sum **A+B** thus obtain a matrix in which each element is given by the sum of the lements of A and B having the same indexes

```
>> A=[1 2 3; -3 -1 2];
>> B=[-1 2 0.5; 3 -3 6];
>> C=A+B

C =

        0    4.0000    3.5000
        0   -4.0000    8.0000
```

Notice: similarly it is possible to compute A-B

If A+k (k real number) is computed, then the number k is added to each element of A

2) PRODUCT BETWEEN A SCALAR AND A MATRIX

A number (scalar) can be multiplied with a matrix by using *

In the resulting matrix each element is multiplied to the given scalar.

```
>> A=[1 2 3; -3 -1 2];
>> B=2*A

B =

     2     4     6
    -6    -2     4
```

EX 12

1) Save the 3×5 matrix A with:

- first row: vector of equally spaced elements from 0 to 12

- second row: all elements equal to 1

- third row: equally spaced elements from 2 to -2

2) save the number b=ln(2) and compute the scalar product between b and A thus obtaining B

3) Compute C by summing A and B

4) Compute D=A-2C+1

3) PUNCTUAL PRODUCT AND PUNCTUAL DIVISION

Consider two matrices A and B having the same dimension. It is possible the **punctual product (and punctual division)** by using **.*** (**./**)

```
>> A=[0 3; -4 2];
>> B=[7 2; -3 -1];
>> C=A.*B

C =

     0     6
    12    -2

>> D=A./B

D =

         0    1.5000
    1.3333   -2.0000
```

1) The element (i,j) of C is given by the product between the element (i,j) of matrices A and B
2) The element (i,j) of D is given by the division between the element (i,j) of matrice A over B.

Notice that Inf or NaN will be notified if the division is not possible in set R.

4) PUNCTUAL POWER

Consider a matrix A and a real number a. then it is possible to compute the power-a of each element of A by using **.^**

```
>> A=[1 2; 3 4];
>> B=A.^2

B =

     1     4
     9    16
```

Notice: Do not forget the point, in fact A^2 gives a different result!

```
>> C=A^2

C =

     7    10
    15    22
```

Similarly each element of a matrix A can be elevated to the corresponding element of a matrix B having the same dimension:

A.^B

```
>> A=[1 2; 3 4];
>> B=[5 3; 3 2];
>> C=A.^B

C =

     1     8
    27    16
```

Observe: here C(i,j)=A(i,j)^B(i,j)

4) FUNCTION APPLIED TO A MATRIX

Consider a matrix A and a function y=f(x). Then it is possible to calculate the **value associated by function f to each element of matrix A**, thus obtaining a matrix having the same dimension of A.

EX: Consider the following matrix

$$A = \begin{pmatrix} 1 & -1 & 0.5 \\ 1 & 0 & 1 \\ 3 & 2 & -4 \end{pmatrix}$$

and function $f(x)=\ln(x^2+4)/(x^3-8)$.

Obtain B by applying function f to matrix A.

Notice: Use correctly the syntax for elementary functions! Do not forget the punctual operators!

```
>> A=[1 -1 0.5; 1 0 1; 3 2 -4]

A =

    1.0000   -1.0000    0.5000
    1.0000         0    1.0000
    3.0000    2.0000   -4.0000

>> B=(log(A.^2+4))./(A.^3-8)

B =

   -0.2299   -0.1788   -0.1837
   -0.2299   -0.1733   -0.2299
    0.1350       Inf   -0.0416
```

Notice: B(3,2) is not a number since the division by 0 has been met.

EX 13

1) Save the following Matrix A 3x3:

- first column: vector of 3 equally spaced elements from 1 to -5
- second column: vector of elements 2,4,5
- third column: elements 1, ln(4), e^5

2) Obtain matrix B by trasposing matrix A

3) Calculate matrix C by elevating each element of matrix A to the correspondent element of matrix B

4) Compute D=-3A+2B/C

5) Let $f(x) = \dfrac{\sqrt{x-1}}{\ln|x+3|}$ obtain matrix E by applying function f to matrix D

Homeworks

1.5

Save the following vectors:

A is a row vector with elements 1,4,7,2

B is a row vector with 4 equally spaced elements from 7 to -5

C is a row vector with elements 0,½,7.8,-3

A. Construct matrix X whose rows are the vectors A, B, C

B. Obtain Y by trasposing X and change the element (2,1) with 0

C. Obtain Z by extracting the matrix having the first two rows and the first two columns from Y

D. Obtain W by extracting the matrix having the last two rows and the last two columns from X

E. Calculate $U=2W+Z^2-\sqrt{W}$

Notice: use the punctual operators when it is necessary!

Homeworks

1.6 Save the matrices:

$$A=\begin{pmatrix}1 & e^{0.5} & 0\\ 3 & 4 & 0.2\end{pmatrix}, B=\begin{pmatrix}0 & 1\\ 3 & -2\\ \ln 5 & 4\end{pmatrix}$$

A. Which is the dimension of A? And of B?

B. Substitute the element (2,2) of A with -3

C. Delete the second column of A and the first row of B

D. Compute C=3A-2B+6

E. Let $f(x)=\left(\sqrt[3]{x}+1\right)^2$, calculate $D=f(C)$

Homeworks

1.7 Create the column vector X with 7 equally speced elements from 10 to 25, the column vector Y with equally speced elements from -10 to 50 step 10, the column vector Z with 7 elements all equal to 2

A. Create matrix A having columns X,Y and Z

B. Compute $B=A^{0.5}$

C. Observe the elements of B, are all real numbers? Why?

D. Calculate the punctual product $C = AB$ and the punctual division D=1/A.

E. Observe the elements of D, are all real numbers? Why?

F. Let $f(x) = |\ln(x+1)^2|$, calculate $f(A)$

Homeworks

1.8 Save the following matrices

$$A = \begin{pmatrix} 3 & -1 \\ 0 & 4 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 \\ 3 & -1 \\ \sqrt{3} & 0 \end{pmatrix}$$

A. Is it possible to compute A+B? Why?

B. Delete from B the second row. Now, is it possible to compute C=A+B?

C. Traspose matrix A and then change the element (1,1) with -3

D. Compute $D=(3A)/(e^2B)$