University of Macerata
Mathematical Methods for Economics and Finance

Exam

May 22, 2024

Surname:
Name:
Student number:

Please, save your work in Matlab in a file entitled:

YourSurname_YourStudentNumber.txt

and send it to mauromaria.baldi@unimc.it by the end of the exam. In the body of the same email, please copy and paste your Matlab work.

Don't forget to write your surname, name and student number on each sheet you are submitting.

1. Financial mathematics:

   A Consider 1000€disposable at time $t = 2$ years and a monthly interest rate of 0.5% with the exponential interest rule. Determine the equivalent money at times $t = 1.5$ years and $t = 6$ years.

   B A project requires an initial cash outlay of 2000€and is expected to generate 800€at the end of year 1 and 1600€at the end of year 2, at which time the project terminates. Calculate the IRR of the project (analitically).

2. In Matlab:

   - Save the matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & e^0.5 & 0 \\ 3 & 4 & 0.2 \end{bmatrix}$$

   and

$$\mathbf{B} = \begin{bmatrix} 0 & 1 \\ 3 & -2 \\ \ln 5 & 4 \end{bmatrix}$$

- Which is the dimension of $\mathbf{A}$? And of $\mathbf{B}$?
- Substitute the element (2, 2) of $\mathbf{A}$ with -3.
- Delete the second column of $\mathbf{A}$ and the first row of $\mathbf{B}$.
- Is it possible to compute the following expression? $\mathbf{C} = 3\mathbf{A} - 2\mathbf{B} + 6$
- Let $f(x) = (\sqrt[3]{x} + 1)^2$. Plot a graph of f and then calculate $\mathbf{D} = f(\mathbf{B})$.

3. Consider the following quadratic form:

$$q(x_1, x_2) = x_1^2 + x_2^2 - 4x_1x_2.$$

- Plot the quadratic form in Matlab
- Consider the (symmetric) matrix $\mathbf{A}$ associated with the quadratic form and find its eigenvalues
- For each eigenvalue, find an associated eigenvector and then normalize it
- Is there an alternative way to compute the eigenvalues? If so, please explain.

4. Consider the function $f(x, y) = x^3+y^3-3x-12y+20$. Find its domain and its critical points. After that, classify all the critical points.

5. (Optional question): state and prove a criterion for the positive and negative definiteness for a 2×2 matrix.