Consider 1000 € disposable at time $t = 2$ years and a monthly interest rate of 0.5% with the exponential interest rule. Determine the equivalent of money at times $t = 1.5$ years and $t = 6$ years.

1 year = 12 months , 2 years = 24 months

1.5 years = 18 months

6 years = $6 \cdot 12$ month = $6 \cdot 6 \cdot 2$ months = $36 \cdot 2$ months =
= 72 months

$$w(2) = w(0)\,(1+i_{12})^{24}$$

$$w(0) = w(2)(1+i_{12})^{-24} = 1000\,€\left(1+\frac{0.05}{100}\right)^{-24} = 988.07\,€$$

$$w(1.5) = w(0)(1+i_{12})^{18} = 988.07\,€\left(1+\frac{0.05}{100}\right)^{18} = 997.01\,€$$

$$w(6) = w(0)(1+i_{12})^{72} = 988.07\,€\left(1+\frac{0.05}{100}\right)^{72} = 1024.28\,€$$

A project requires an initial cash outlay of 2000 € and is expected to generate 800 € at the end of year 1 and 1600 € at the end of year 2, at which time the project will terminate. Calculate the IRR of the project (analitically).

$$OF = \{(-2000,\ 800,\ 1600),\ (0,1,2)\}$$

$$-2000 + 800(1+i)^{-1} + 1600(1+i)^{-2} = 0$$

$$v := (1+i)^{-1}$$

$$-2000 + 800v + 1600v^2 = 0$$

$$16v^2 + 8v - 20 = 0$$

$$4v^2 + 2v - 5 = 0$$

$$v = \frac{-1 \pm \sqrt{21}}{4}$$ the negative root is discarded

$$v = \frac{-1+\sqrt{21}}{4}$$

$$(1+i)^{-1} = \frac{-1+\sqrt{21}}{4}$$

$$1+i = \frac{4}{\sqrt{21}-1}$$

$$i = \frac{4}{\sqrt{21}-1} - 1 \cong 0.12 = 12\%$$

$f(x,y) = x^3 + y^3 - 3x - 12y + 20$

$\operatorname{dom} f = \mathbb{R}^2$

$f'_x = 3x^2 - 3 \qquad f'_y = 3y^2 - 12$

$\begin{cases} 3x^2 - 3 = 0 \\ 3y^2 - 12 = 0 \end{cases} \qquad \begin{cases} x^2 - 1 = 0 \\ y^2 - 4 = 0 \end{cases} \qquad \begin{cases} x = \pm 1 \\ y = \pm 2 \end{cases}$

$A(1,2)$; $B(1,-2)$; $C(-1,2)$; $D(-1,-2)$

$f''_{xx} = 6x$; $f''_{xy} = f''_{yx} = 0$; $f''_{yy} = 6y$

$$\underline{\underline{H}}_f(x,y) = \begin{bmatrix} 6x & 0 \\ 0 & 6y \end{bmatrix}$$

$\underline{\underline{H}}_f(A) = \begin{bmatrix} 6 & 0 \\ 0 & 12 \end{bmatrix}$ positive definite
A is a relative minimum

$\underline{\underline{H}}_f(B) = \begin{bmatrix} 6 & 0 \\ 0 & -12 \end{bmatrix}$ indefinite
B is a saddle point

$\underline{\underline{H}}_f(C) = \begin{bmatrix} -6 & 0 \\ 0 & 12 \end{bmatrix}$ indefinite
C is a saddle point

$\underline{\underline{H}}_f(D) = \begin{bmatrix} -6 & 0 \\ 0 & -12 \end{bmatrix}$ negative definite
D is a relative maximum.

Plot in Matlab the following quadratic form and study it analitically:

$$q(x_1, x_2) = x_1^2 + x_2^2 - 4x_1x_2$$

$$q(x_1, x_2) = x_1^2 + x_2^2 - 2x_1x_2 - 2x_2x_1 =$$

$$= \underbrace{[x_1 \quad x_2]}_{\underline{x}^T} \underbrace{\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}}_{\underline{\underline{A}}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\underline{x}}$$

$$\det(\underline{\underline{A}} - \lambda \underline{\underline{I}}) = 0$$

$$\begin{vmatrix} 1-\lambda & -2 \\ -2 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)^2 - 4 = 0$$

$$(\lambda - 1)^2 = 4$$

$$\lambda - 1 = \pm 2 \quad ; \quad \lambda = 1 \pm 2 \begin{cases} -1 \\ 3 \end{cases}$$

The quadratic form is indefinite

On:

$$\underline{\underline{A}} = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$$

$|A_1| = 1 > 0$

$|A_2| = \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = 1-4 = -3 < 0$

Thus, the matrix $\underline{\underline{A}}$ is indefinite and so is the associated quadratic form.

$$(\underline{\underline{A}} - \lambda \underline{\underline{I}})\underline{v} = \underline{0}$$

$\underline{\lambda = -1:}$

$(\underline{\underline{A}} + \underline{\underline{I}})\underline{v} = \underline{0}$

$$\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 2 & -2 \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \qquad \begin{aligned} x - y &= 0; \\ y &= x \end{aligned}$$

If $x = 1$, then $y = 1$

$\underline{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector associated to the eigenvalue $\lambda = 1$

$\|\underline{v}\| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\hat{\underline{v}} = \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]^T$ is a normalized eigenvector associated to the eigenvalue $\lambda = 1$

$\underline{\lambda = 3:}$

$(\underline{\underline{A}} - 3\underline{\underline{I}})\, \underline{v} = \underline{0}$

$\begin{bmatrix} -2 & -2 \\ -2 & -2 \end{bmatrix} \sim \begin{bmatrix} -2 & -2 \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ $\quad x + y = 0$

$y = -x$

If $x = 1$, then $y = -1$

$\underline{v} = (1, -1)^T$ is an eigenvector associated to the eigenvalue $\lambda = 3$

$\| \underline{v} \| = \sqrt{2}$

$\hat{\underline{v}} = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right)^T$ is a normalized eigenvector associated to the eigenvalue $\lambda = 3$.