

University of Macerata
Mathematical Methods for Economics and Finance

Exam

June 17, 2024

Surname:

Name:

Student number:

Please, save your work in Matlab in a file entitled:

YourSurname_YourStudentNumber.txt

and send it to mauromaria.baldi@unimc.it by the end of the exam. In the body of the same email, please copy and paste your Matlab work.

Don't forget to write your surname, name and student number on each sheet you are submitting.

1. Let be given 5000 euro today to be invested for 15 months, with the simple interest rule and $i = 9\%$ or with the exponential interest rule and $i_2 = 4\%$. Which is the most convenient option?
2. Save the following vectors in Matlab:
 - **a** is a row vector with elements 1, 4, 7, and 2.
 - **b** is a row vector with 4 equally spaced elements from 7 to -5.
 - **c** is a row vector with elements 0, 1/2, 7.8, and -3.

Then:

- Construct the matrix **X** whose rows are the vectors **a**, **b**, and **c**.
- Obtain **Y** by trasposing **X** and then change the element in position (2,1) with 0.
- Obtain **Z** by extracting the matrix having the first two rows and the first two columns from **Y**.
- Obtain **W** by extracting the matrix having the last two rows and the last two columns from **X**.
- Calculate $\mathbf{U} = 2\mathbf{W} + \mathbf{Z}^2 - \sqrt{\mathbf{W}}$.

3. Sketch the domain of the following function:

$$f(x, y) = \frac{\log_{10}(4 - x^2 - y^2)}{\sqrt[3]{y - x}} - \sqrt{x^2 + y^2 - 1}.$$

4. Consider the following matrix \mathbf{A} :

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

- Compute the eigenvalues of \mathbf{A} (analytically).
 - Find an eigenvector of at least one eigenvalue.
 - Normalize the chosen eigenvector(s).
 - Find the quadratic form associated to the matrix \mathbf{A} .
5. Solve the following constrained maximization problem: $\max f(x, y) = 8x^2 - 2y$ subject to the constraint $x^2 + y^2 = 1$.
6. Optional question: provide a sketch of the proof of the Hessian test.