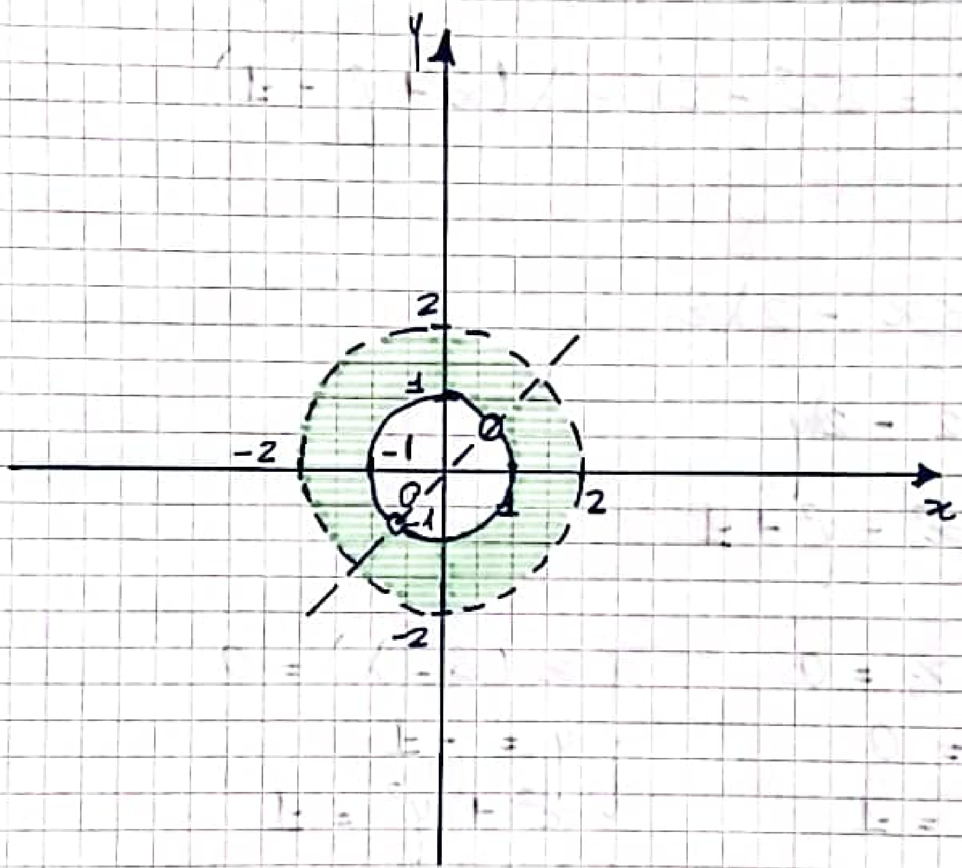
$$f(x,y) = \frac{\log_{10}(4-x^2-y^2)}{\sqrt[3]{y-x}} + \sqrt{x^2+y^2-1}$$

$$\begin{cases} 4-x^2-y^2 > 0 \\ y-x \neq 0 \\ x^2+y^2-1 \geq 0 \end{cases} \qquad \begin{cases} x^2+y^2 < 4 \\ y \neq x \\ x^2+y^2 \geq 1 \end{cases}$$



$\max \quad 8x^2 - 2y$

s.t.: $\quad x^2 + y^2 = 1$

$f(x, y) = 8x^2 - 2y$

$h(x, y) = x^2 + y^2$

$\nabla h \neq \underline{0} \quad ; \quad (2x, 2y) \neq (0,0) \; ; \; (x, y) \neq (0,0)$.

This point does not lie in the constraint set.

Thus, qualification constraints are always met.

$\mathcal{L}(x, y, \lambda) = 8x^2 - 2y - \lambda(x^2 + y^2 - 1)$

$\nabla \mathcal{L} = \underline{0}$

$\frac{\partial \mathcal{L}}{\partial x} = 16x - 2\lambda x$

$\frac{\partial \mathcal{L}}{\partial y} = -2 - 2\lambda y$

$\frac{\partial \mathcal{L}}{\partial \lambda} = -x^2 - y^2 + 1$

$$\begin{cases} 8x - \lambda x = 0 \\ 1 + \lambda y = 0 \\ x^2 + y^2 = 1 \end{cases} \qquad \begin{cases} x(8 - \lambda) = 0 \\ 1 + \lambda y = 0 \\ x^2 + y^2 = 1 \end{cases}$$

$$\begin{cases} x = 0 \\ y = -1/\lambda, \quad \lambda \neq 0 \\ \frac{1}{\lambda^2} = 1 \end{cases} \qquad \begin{cases} x = 0 \\ y = \mp 1 \\ \lambda = \pm 1 \end{cases} \qquad \begin{matrix} A(0, -1) \\ B(0, 1) \end{matrix}$$

or

$$\begin{cases} \lambda = 8 \\ 1 + 8y = 0 \\ x^2 + y^2 = 1 \end{cases} \qquad \begin{cases} \lambda = 8 \\ y = -\frac{1}{8} \\ x^2 = 1 - \frac{1}{64} = \frac{63}{64} \end{cases}$$

$$\begin{cases} \lambda = 8 \\ y = -\frac{1}{8} \\ x = \pm \frac{3}{8}\sqrt{7} \end{cases} \qquad C\left(-\frac{3}{8}\sqrt{7}, -\frac{1}{8}\right); \; D\left(\frac{3}{8}\sqrt{7}, \frac{1}{8}\right)$$

$f(x,y) = 8x^2 - 2y$

The constraint set

$S = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ is compact and the objective function f is continuous. Thus, by Weierstrass' theorem, there exists a global maximum (and a global minimum).

We look for it among all the critical points:

$f(A) = f(0,-1) = 8 \cdot 0^2 - 2(-1) = 2$

$f(B) = f(0,1) = 8 \cdot 0^2 - 2 \cdot 1 = -2$

$f(C) = f\left(-\frac{3}{8}\sqrt{7}, -\frac{1}{8}\right) = 8 \cdot \left(-\frac{3}{8}\sqrt{7}\right)^2 - 2\left(-\frac{1}{8}\right) =$

$= 8 \cdot \frac{9}{64} \cdot 7 + \frac{1}{4} = \frac{9 \cdot 7}{8} + \frac{2}{8} = \frac{63}{8} + \frac{2}{8} =$

$= \frac{65}{8}$

$f(D) = f\left(\frac{3}{8}\sqrt{7}, \frac{1}{8}\right) = 8 \cdot \left(\frac{3}{8}\sqrt{7}\right)^2 - 2 \cdot \frac{1}{8} =$

$= 8 \cdot \frac{9}{8^2} \cdot 7 - \frac{2}{8} = \frac{9 \cdot 7}{8} - \frac{2}{8} = \frac{63}{8} - \frac{2}{8} = \frac{61}{8}$

Therefore, C is the global maximum and B is the global minimum

$$\underline{\underline{A}} \quad \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Eigenvalues

$$\det(\underline{\underline{A}} - \lambda \underline{\underline{I}}) = 0$$

$$\begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$-\lambda \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & -\lambda \end{vmatrix} + \begin{vmatrix} 1 & -\lambda \\ 1 & 1 \end{vmatrix} = 0$$

$$-\lambda(\lambda^2 - 1) - (-\lambda - 1) + 1 + \lambda = 0$$

$$-\lambda(\lambda + 1)(\lambda - 1) + \lambda + 1 + 1 + \lambda = 0$$

$$-\lambda(\lambda + 1)(\lambda - 1) + 2\lambda + 2 = 0$$

$$-\lambda(\lambda + 1)(\lambda - 1) + 2(\lambda + 1) = 0$$

$$(\lambda + 1)\,[-\lambda(\lambda - 1) + 2] = 0$$

$$(\lambda + 1)(-\lambda^2 + \lambda + 2) = 0$$

$$(\lambda + 1)(\lambda^2 - \lambda - 2) = 0$$

$$(\lambda + 1)(\lambda - 2)(\lambda + 1) = 0$$

$$(\lambda + 1)^2(\lambda - 2) = 0$$

$\lambda = -1$ Y $\lambda = 2$

Eigenvectors

$\lambda = -1$

$$(\underline{\underline{A}} + \underline{\underline{I}})\,\underline{v} = 0 \quad , \quad \underline{v} = (x, y, z)^T$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x + y + z = 0$$

$$z = -x - y$$

$$\underline{v} = \begin{bmatrix} x \\ y \\ -x-y \end{bmatrix} = x \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$\underline{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad , \quad \underline{v}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$\| \underline{v}_1 \| = \sqrt{1^2 + 0^2 + (-1)^2} = \sqrt{2}$$

$$\hat{\underline{v}}_1 = \left(\frac{1}{\sqrt{2}} , 0 , -\frac{1}{\sqrt{2}} \right)^T$$

$$\| \underline{v}_2 \| = \sqrt{0^2 + 1^2 + (-1)^2} = \sqrt{2}$$

$$\hat{\underline{v}}_2 = \left(0 , \frac{1}{\sqrt{2}} , -\frac{1}{\sqrt{2}} \right)^T$$

$\underline{\lambda = 2}$

$$(\underline{\underline{A}} - 2\underline{\underline{I}})\underline{v} = \underline{0}$$

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} -2 & 1 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix} \sim$$

$$\begin{bmatrix} -2 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} -2x + y + z = 0 \\ y - z = 0 \end{cases} \qquad \begin{cases} -2x + 2z = 0 \\ y = z \end{cases}$$

$$\begin{cases} x = z \\ y = z \end{cases}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} z \\ z \\ z \end{bmatrix} = z \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\underline{v_3} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\| \underline{v_3} \| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$\underline{\hat{v}_3} = \frac{\underline{v_3}}{\| \underline{v_3} \|} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)^T$$

$W_0 = 5000\ €$

Simple interest rule:

$i = 9\%$

$m\, i_m = n\, i_n$

$12 \cdot i_{12} = 1 \cdot i$

$i_{12} = \frac{1}{12}\, i = \frac{1}{12} \cdot 0.09 = 0.0075$

$W^s_{15} = W_0 \left(1 + i_{12}\, t\right) =$

$= 5000\ € \left(1 + 0.0075 \cdot 15\right) = 5562{,}5\ €$

Compound interest rule:

$i_2 = 4\%$

$(1 + i_m)^m = (1 + i_n)^n$

$1 + i_m = (1 + i_n)^{n/m}$

$i_m = (1 + i_n)^{n/m} - 1$

$i_{12} = (1 + i_2)^{2/12} - 1 = (1.04)^{1/6} - 1 = 0.0066$

$W^c_{15} = W_0 \left(1 + i_{12}\right)^t = 5000€ \left(1.0066\right)^{15} = 5518.5\ €$

$$\underline{\underline{x}} = [x_1, x_2, x_3]^T$$

$$\underline{\underline{x}}^T \underline{\underline{A}}\, \underline{\underline{x}} = [x_1 \quad x_2 \quad x_3] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =$$

$$= [x_1 \quad x_2 \quad x_3] \begin{bmatrix} x_2 + x_3 \\ x_1 + x_3 \\ x_1 + x_2 \end{bmatrix} =$$

$$= x_1 (x_2 + x_3) + x_2 (x_1 + x_3) + x_3 (x_1 + x_2) =$$

$$= \underline{x_1 x_2} + \underline{\underline{x_1 x_3}} + \underline{x_2 x_1} + x_2 x_3 + \underline{\underline{x_3 x_1}} + x_3 x_2 =$$

$$= 2 x_1 x_2 + 2 x_1 x_3 + 2 x_2 x_3$$