1

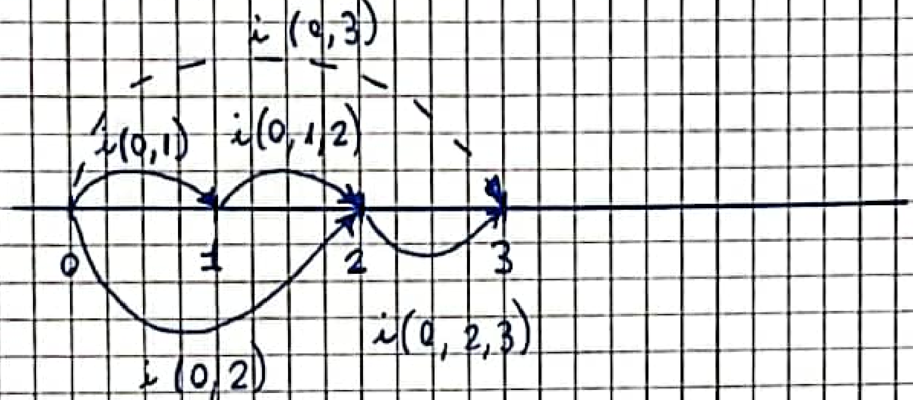
$i(0,1) = 0.05$

$i(0,2) = 0.07$

$i(0,2,3) = 0.06$

$i(0,1,2) = ?$

$i(0,3) = ?$



$$\cancel{W_0}\,(1+i(0,1))^1\,(1+i(0,1,2))^1 = \cancel{W_0}\,(1+i(0,2))^2$$

$$1+i(0,1,2) = \frac{(1+i(0,2))^2}{1+i(0,1)}$$

$$i(0,1,2) = \frac{(1+i(0,2))^2}{1+i(0,1)} - 1 =$$

$$= \frac{(1+0.07)^2}{1+0.05} - 1 = 0.09$$

$$\cancel{W_0}\,(1+i(0,2))^2\,(1+i(0,2,3))^1 = \cancel{W_0}\,(1+i(0,3))^3$$

$$i(0,3) = \sqrt[3]{(1+i(0,2))^2\,(1+i(0,2,3))} - 1 =$$

$$= \sqrt[3]{(1+0.07)^2\,(1+0.06)} - 1 = 0.067$$

(3) $z = \dfrac{x}{y-1}$

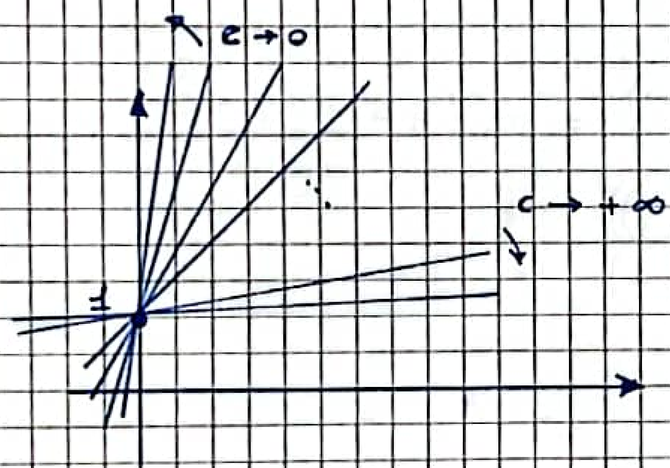
$y \neq 1$

$\dfrac{x}{y-1} = c$

$x = c(y-1)$

$y - 1 = \dfrac{x}{c}$

$y = \dfrac{x}{c} + 1$



(4) $f(x,y) = x^3 + y^3 + 3xy$

$f'_x = 3x^2 + 3y = 0$
$f'_y = 3y^2 + 3x = 0$

$\begin{cases} x^2 + y = 0 \\ y^2 + x = 0 \end{cases}$

$\begin{cases} y = -x^2 \\ x^4 + x = 0 \end{cases}$ $\begin{cases} y = -x^2 \\ x(x^3+1) = 0 \end{cases}$

$\begin{cases} y = 0 \\ x = 0 \end{cases} \vee \begin{cases} y = -1 \\ x = -1 \end{cases}$ $A(0,0)$ e $B(-1,-1)$

$f''_{xy} = 3 = f''_{yx}$; $f''_{xx} = 6x$; $f''_{yy} = 6y$

$$\underline{\underline{H}}f(x,y) = \begin{bmatrix} 6x & 3 \\ 3 & 6y \end{bmatrix}$$

$$\underline{\underline{H}}f(0,0) = \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix} \text{ indefinite.}$$

The origin is a saddle point.

$$\underline{\underline{H}}f(-1,-1) = \begin{bmatrix} -6 & 3 \\ 3 & -6 \end{bmatrix} < 0$$

Then $(-1,-1)$ is a local maximum.

(5) $f(x,y) = xe^y + 1$, $P_0(1,0)$

$f'_x = e^y$, $f'_y = xe^y$

$$\nabla f(x,y) = \begin{bmatrix} e^y \\ xe^y \end{bmatrix}$$

$$\nabla f(x_0,y_0) = \nabla f(1,0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$f''_{xx} = 0$, $f''_{xy} = e^y = f''_{yx}$, $f''_{yy} = xe^y$

$$\underline{\underline{H}}f(x,y) = \begin{bmatrix} 0 & e^y \\ e^y & xe^y \end{bmatrix} \qquad \underline{\underline{H}}f(x_0,y_0) = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$T_0(x, y) = f(x_0, y_0) = f(1, 0) = 2$$

$$T_1(x, y) = f(x_0, y_0) + \nabla f^T(x_0, y_0)(\underline{x} - \underline{x}_0) =$$

$$= 2 + \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x - 1 \\ y \end{bmatrix} = 2 + x - 1 + y =$$

$$= 1 + x + y$$

$$T_2(x, y) = f(x_0, y_0) + \nabla f^T(x_0, y_0)(\underline{x} - \underline{x}_0) +$$

$$+ \frac{1}{2}(\underline{x} - \underline{x}_0)^T \underline{\underline{H}}f(\underline{x}_0)(\underline{x} - \underline{x}_0) =$$

$$= 1 + x + y + \frac{1}{2} \begin{bmatrix} x - 1 & y \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x - 1 \\ y \end{bmatrix} =$$

$$= 1 + x + y + \frac{1}{2} \begin{bmatrix} y & x - 1 + y \end{bmatrix} \begin{bmatrix} x - 1 \\ y \end{bmatrix} =$$

$$= 1 + x + y + \frac{1}{2} \left[y(x - 1) + y(x - 1) + y^2 \right] =$$

$$= 1 + x + \cancel{y} + xy - \cancel{y} + \frac{1}{2} y^2$$

$$T_2(x, y) = 1 + x + xy + \frac{1}{2} y^2$$