

WELCOME

time
 t_0

time
 t_1

$t_1 \geq t_0$

$w(t_0)$: amount of money at time t_0

$t_0 = 0$ now

$t_1 = 2$ 2 years

If now you have 500 €,

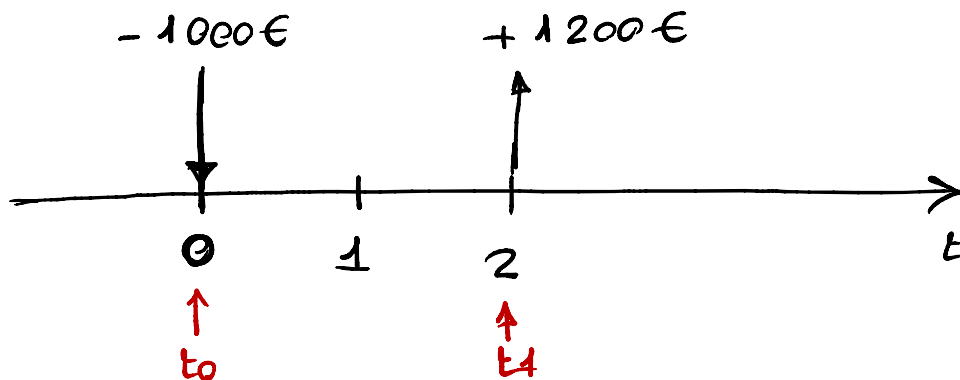
$$w(0) = 500 \text{ €}$$

$\uparrow$
 $t_0 = 0$

2 main operations:

1) investment

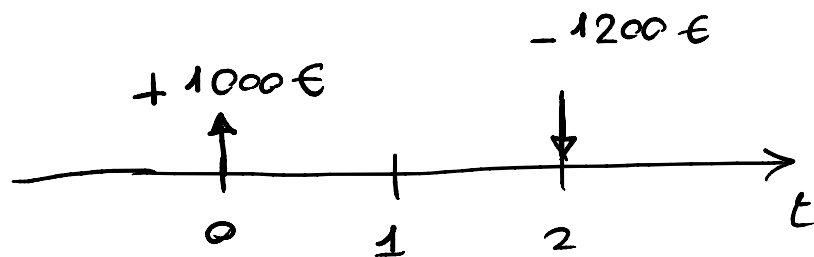
You invest 1000 € now and after 2 years you collect 1200 €



OF : financial operation

$$OF = \{ \overset{\text{exit}}{(-1000, +1200)}; \overset{\text{entrance}}{(0, 2)} \}$$

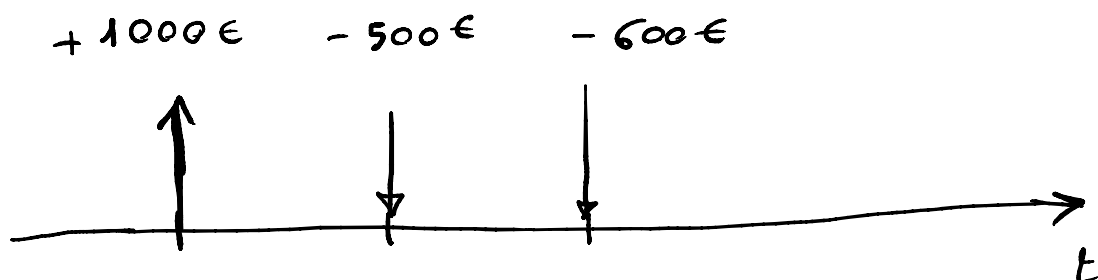
2) borrow

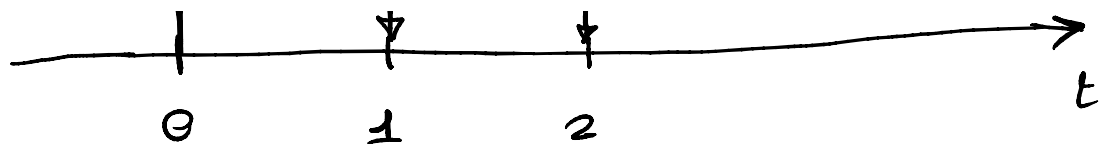


$$OF2 = \{ \overset{\text{entrance}}{(+1000\text{€}, -1200\text{€})}; \overset{\text{exit}}{(0, 2)} \}$$

So far, we have dealt with
SIMPLE FINANCIAL OPERATIONS, i.e., with
one entry and one exit

COMPLEX FINANCIAL OPERATIONS: they are
composed by more than two amounts.





$$OF3 = \left\{ (+1000\text{€}, -500\text{€}, -600\text{€}), (0, 1, 2) \right\}$$

FINANCIAL LAWS

- 1) rule of simple interest
- 2) rule of compound interest

SIMPLE INTEREST

$t = 0$ now

$w(0)$: amount of money now,
also called the principal

The amount of money we will collect is
called the future value

If I decide to collect the money after
1 year :

↓ interest rate

+ year.

$$w(\underline{1}) = w(0) + \overset{\text{interest rate}}{\downarrow} i w(0) = w(0) (1 + \underline{1}i)$$

principal + $\underbrace{\hspace{1cm}}$ fraction of the principal

If I decide to collect the money after 2 years:

$$w(\underline{2}) = \underbrace{w(1)}_{\text{the money at the previous year}} + \underbrace{i w(0)}_{\text{fraction of the principal}} =$$

$$= w(0) + i w(0) + i w(0) = w(0) (1 + \underline{2}i)$$

$$w(\underline{3}) = w(2) + i w(0) = w(0) (1 + 2i) + i w(0) = w(0) (1 + \underline{3}i)$$

$$w(m) = w(0) (1 + m i), \quad m \in \mathbb{N}$$

$$\text{If } m = 3$$

$$w(3) = w(0) (1 + 3i)$$

$$\text{If } m = 2$$

$$w(2) = w(0) (1 + 2i)$$

$$w(t) = w(0) (1 + it)$$

simple interest rule

$$w(\underline{1}) = w(0) + \overset{\downarrow}{i} w(0) = w(0) (1 + \underline{1}i)$$

Suppose you invest $1 \in$ today

Suppose you invest $1 \in$ today

$$w(0) = +1$$

After a time t the future value will be:

$$w(t) = 1 \cdot (1 + it) = \underline{1 + it}$$

General ~~case~~:

$$\frac{w(t)}{w(0)} = \frac{\cancel{w(0)} (1 + it)}{\cancel{w(0)}}$$

$$\frac{w(t)}{w(0)} = \underline{1 + it}$$

the ratio between
the future value
and the principal } GROWTH FACTOR

EX3: Consider the rule of simple interest. If today we invest 2000 euro at an interest rate (annual) of $10\% = 0.1$, then in 3 years we will receive

$$w(t) = w(0) (1 + it)$$

$$w(0) = 2000 \in$$

$$t = 3$$

$$w(3) = \in (1 + 0.1 \cdot 3)$$

```
>> 2000*(1 + 0.1*3)
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ans =
```

```
2600
```

or

```
>> 2000*(1 + .1*3)
```

```
ans =
```

```
2600
```

EX4: The life of the investment is 18 months and it is given the annual interest rate $i=10\%$. To correctly find the future value we can proceed as follows:

2000 € to invest

$$w(t) = w(0) (1 + i t)$$

months (pointing to t)
years (pointing to i)
No

$$18 \text{ months} = \frac{18}{12} \text{ years} \quad \text{so} \quad t = \frac{18}{12}$$

After 18 months we collect:

$$2000 \text{ €} \left(1 + 0.1 \cdot \frac{18}{12} \right) = 2300 \text{ €}$$

```
>> 2000*(1 + .1*18/12)
```

```
ans =
```

```
2300
```