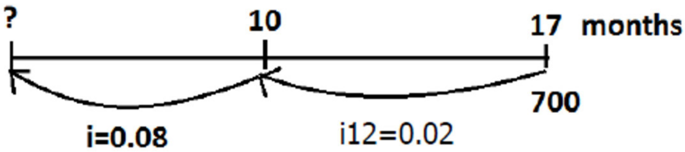
EX10: We will receive 10000 euro in one year and it is given a semiannual interest rate of 0.05 in the exponential interest rule. The present value is:

EX11: The amount of 700 euro will be disposable in 17 months. We wants to determine its present value with the exponential rule by considering that: (1) during the first 10 months it is given i=0.08.(2) during the last period it is applied the monthly interest rate 2%

Graphically:



$$i_2 = 0.05$$

$$w(t) = \underline{w(0)}\,(1+i)^t$$

semesters

$$w(1) = w(0)\,(1+i_2)$$

$$w(2) = w(0)\,(1+i_2)^2$$

$$w(0) = w(t)\,(1+i)^{-t}$$

i (year)

t (years)

$$w(0) = w(t)\,(1+i_2)^{-t}$$

i_2

t in semesters

1 year = 2 semesters

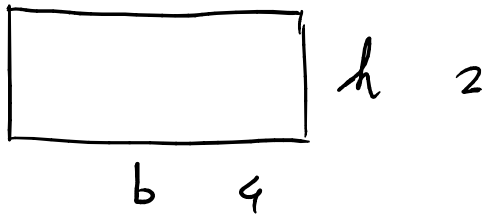
$$w(0) = w(2)\,(1+i_2)^{-2} =$$

$$= 10000\,€\,(1+0.05)^{-2} = 9070.29\,€$$

$$1e4 = 1 \times 10^4$$

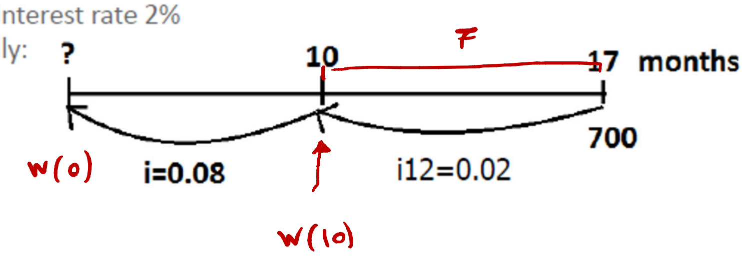
$$0.1 = 10^{-1} = 1e{-1}$$

$$2^3 \qquad 2^\wedge 3$$



EX11: The amount of 700 euro will be disposable in 17 months. We wants to determine its present value with the exponential rule by considering that: (1) during the first 10 months it is given i=0.08.(2) during the last period it is applied the monthly interest rate 2%

Graphically:



i_m : interest rate referred to the m-th part of the year

i_{12} : refers to the 12-th part of the year, i.e. to a month

i_2 : refers to semesters

i_1 : refers to a year, the same as i

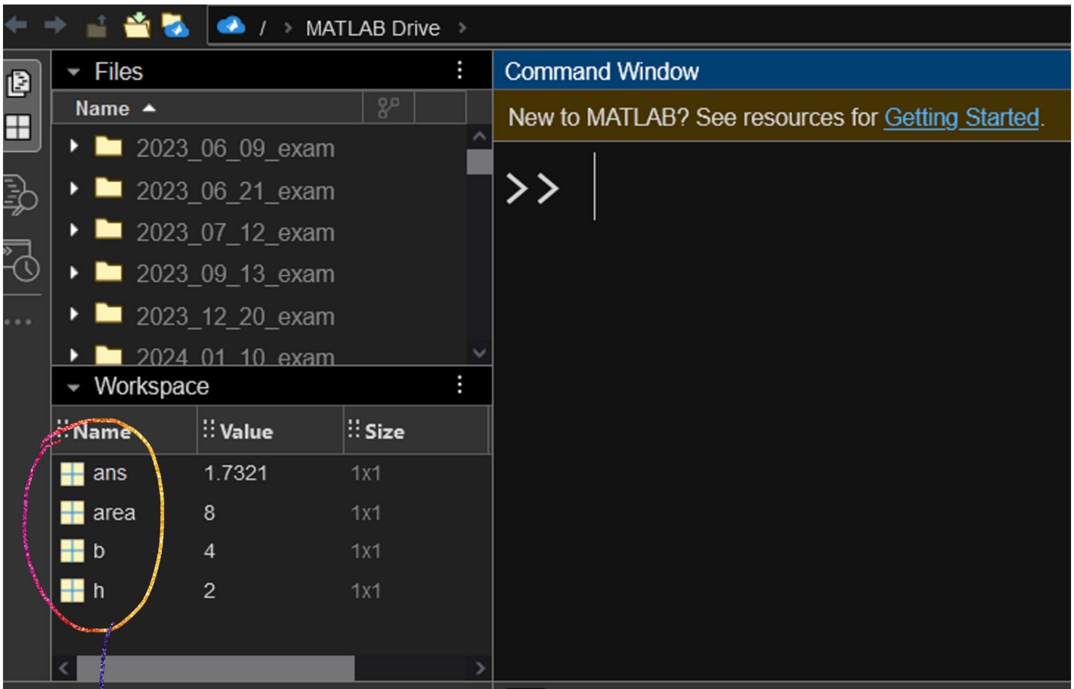
$$w(17) = w(10)\,(1+i_{12})^{7}$$

$$w(10) = w(17)\,(1+i_{12})^{-7}$$

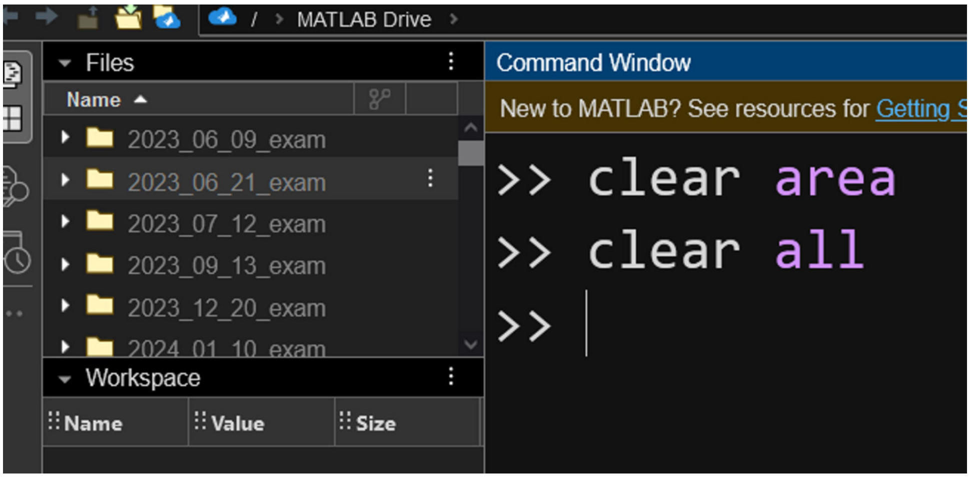
$$w(10) = w(17)(1 + i_{12})^{-7}$$

```
area =

     8

>> % this is a comment
>> % clear the screen
>> clc
```

the result is:



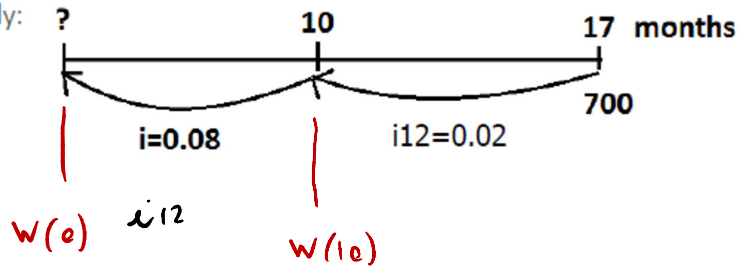
our variables are still present



$$w(10) = w(17)(1+i_{12})^{-7} = 609.39\ €$$

EX11: The amount of 700 euro will be disposable in 17 months. We wants to determine its present value with the exponential rule by considering that: (1) during the first 10 months it is given i=0.08.(2) during the last period it is applied the monthly interest rate 2%

Graphically:



$$i_m = (1+i)^{\frac{1}{m}} - 1$$

$$i_{12} = (1+i)^{\frac{1}{12}} - 1 = 0.0064$$

$\boxed{\cancel{0.02}}$

$i12$

$\boxed{}$

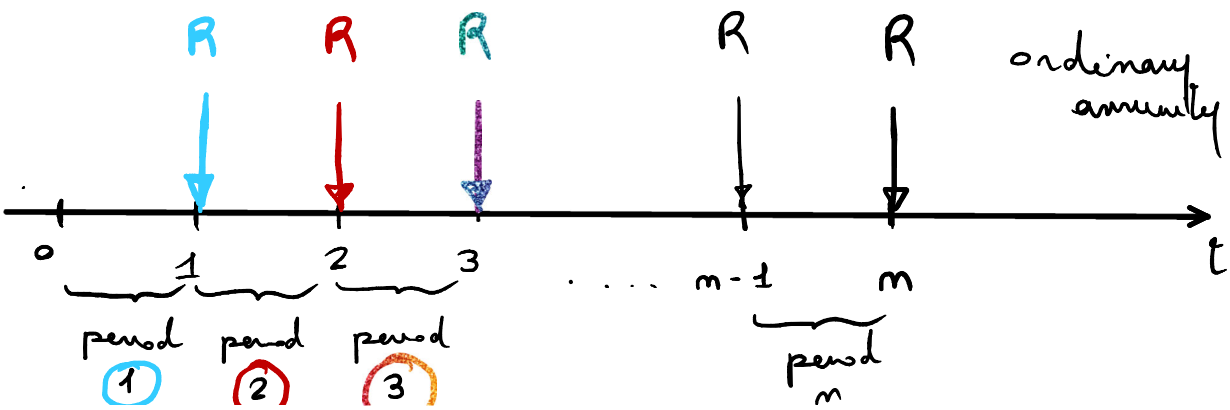
$i12p1$

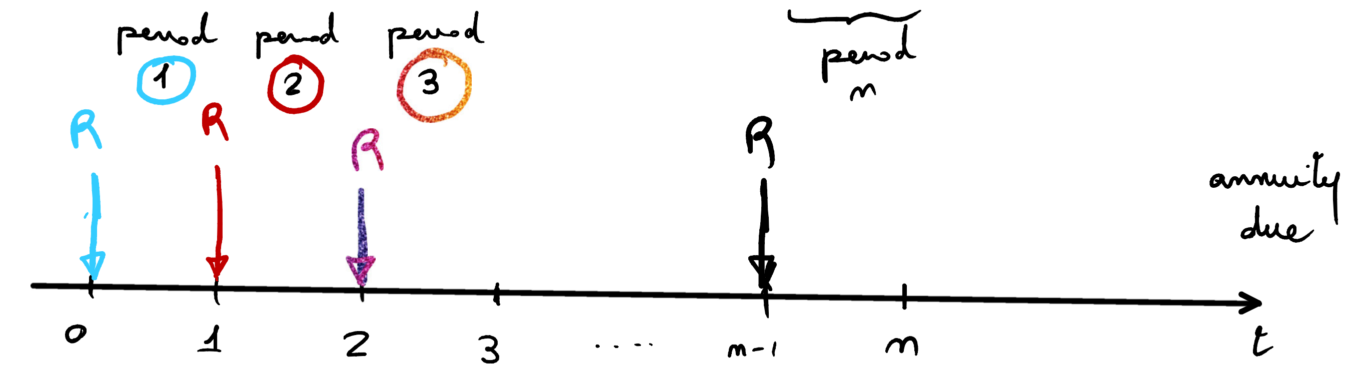
$$w(10) = w(0)(1+i_{12})^{10}$$

$$w(0) = w(10)(1+i_{12})^{-10} = 571.54\ €$$

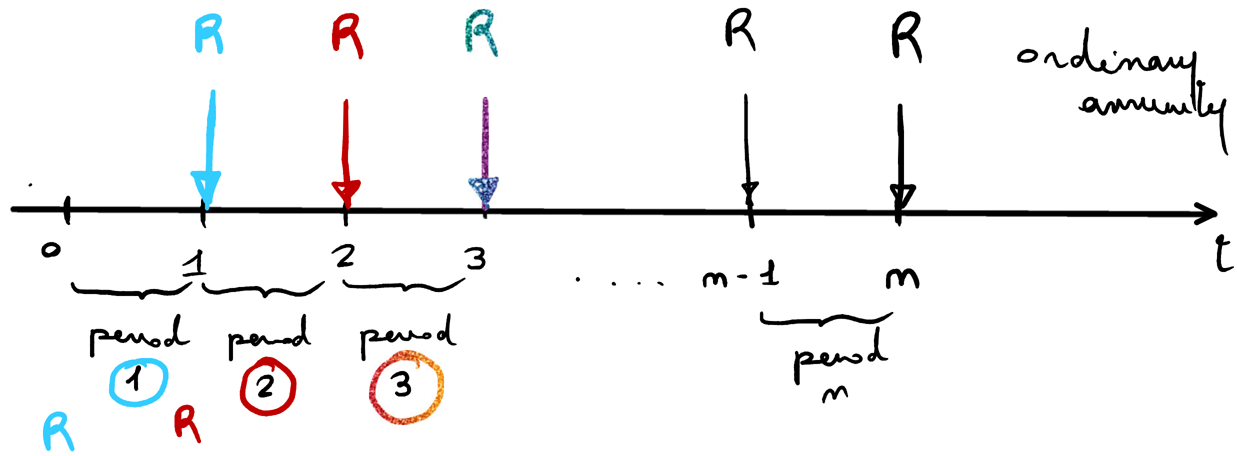
% natural logarithm

```
log(2)

ans =

0.6931


% decimal logarithm
log10(100)

ans =

2


% absolute value
abs(5)

ans =

5


abs(-5)

ans =

5
```





What is the present value of the following annuity?



$W(\underline{R}, 0) = ?$

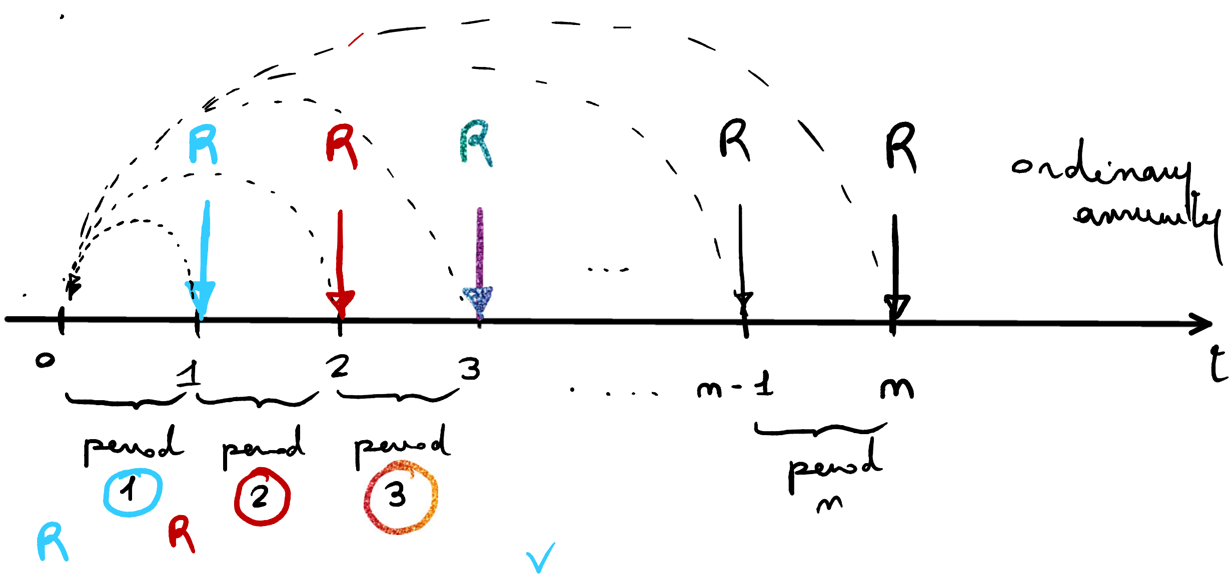
vector

$$[R, R, \ldots\ldots, R]$$

n elements

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$ row vector

$$\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$ column vector



$$W(\underline{R}, 0) = R\overbrace{(1+i)^{-1}}^{v} + R(1+i)^{-2} + R(1+i)^{-3} + \\ + R(1+i)^{-(m-1)} + R(1+i)^{-m} \qquad (*)$$

$$v(t) = (1+i)^{-t}$$ discount factor

Let me call $v = (1+i)^{-1}$

So $(*)$ becomes:

$$W(\underline{R}, 0) = Rv + Rv^2 + Rv^3 + \dots + Rv^{m-1} + \\ + Rv^m =$$

$$= Rv\left(1 + v + v^2 + \dots + v^{m-1}\right)$$

$$S = 1 + x + x^2 + x^3 + \qquad + x^{k-1} + x^k$$

$$S = 1 + \underbrace{x + x^2 + x^3 + \dots + x^{k-1} + x^k}$$

$$S = 1 + x(1 + x + x^2 + \dots + x^{k-2} + x^{k-1})$$

$$S = 1 + x(\underbrace{1 + x + x^2 + \dots + x^{k-2} + x^{k-1} + x^k}_{S} - x^k)$$

$$S = 1 + x(S - x^k)$$

$$S = 1 + xS - x^{k+1}$$

$$S - xS = 1 - x^{k+1}$$

$$S(1-x) = 1 - x^{k+1} \qquad \text{if } x \neq 1 :$$

$$S = \frac{1 - x^{k+1}}{1 - x}$$

$$S = 1 + \underbrace{x + x^2 + x^3 + \dots + x^{k-1} + x^k} =$$

$$= \begin{cases} k+1 & \text{if } x = 1 \\ \dfrac{1 - x^{k+1}}{1 - x} & \text{if } x \neq 1 \end{cases}$$

For example $x = 2$, $k = 3$

$$S = 1 + 2 + 2^2 + 2^3 =$$

$$= 1 + 2 + 4 + 8 = 15$$

$$\frac{1 - 2^4}{1 - 2} = \frac{1 - 16}{-1} = 15$$

$$w(\underline{R}, 0) = R v \left(1 + v + v^2 + \dots + v^{n-1}\right) =$$

$$= R v \frac{1 - v^n}{1 - v}$$

$$v = (1 + i)^{-1}$$

$$v = \frac{1}{1 + i}$$