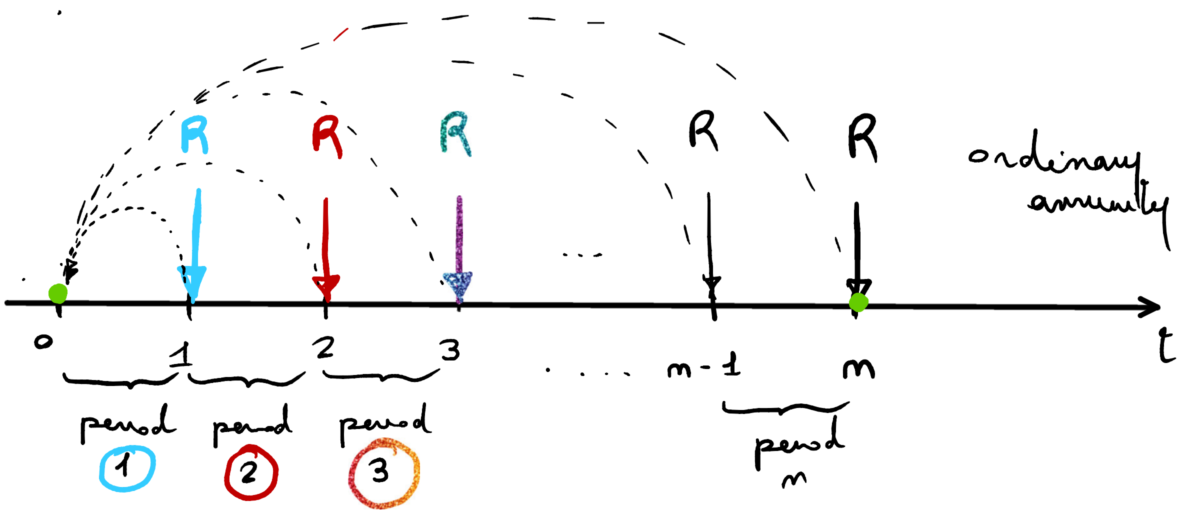


$$w(\underline{R}, 0) = R(1+i)^{-1} + R(1+i)^{-2} + R(1+i)^{-3} +$$

$$+ R(1+i)^{-(n-1)} + R(1+i)^{-n} \qquad (*)$$

$$v := \frac{1}{1+i} = (1+i)^{-1}$$

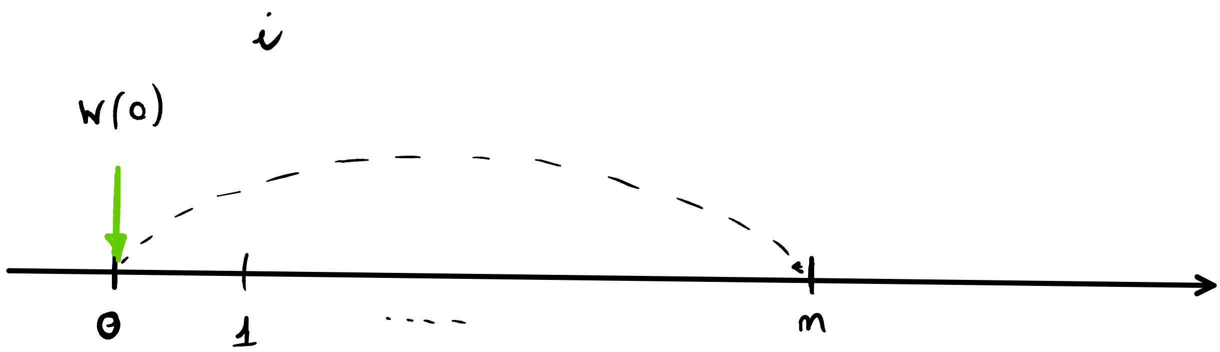
$$w(\underline{R}, 0) = R\, v\, (1 + v + v^2 + \dots + v^{n-1}) =$$

$$= R\cancel{(v)}\frac{\frac{1}{\cancel{v}}(1 - v^n)}{(1-v)\frac{1}{v}} = R\,\frac{1}{\frac{1}{v} - 1}(1 - v^n) =$$

$$= R\,\frac{1}{\cancel{1} + i - \cancel{1}}\left(1 - \left((1+i)^{-1}\right)^n\right) =$$

$$= R\,\underbrace{\frac{1 - (1+i)^{-n}}{i}}_{a_{\overline{n}|i}} = R\, a_{\overline{n}|i}$$

$a_{\overline{n}|i}$ a angle n at i



$$W(m) = W(0)\cdot(1+i)^{m}$$

So the value of the annuity after m years is:

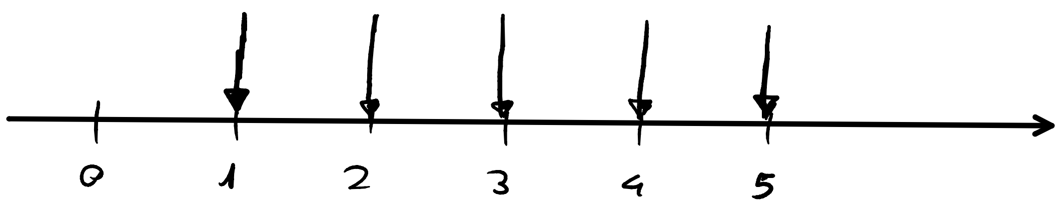
$$W(\underline{R}, m) = W(\underline{R}, 0)\,(1+i)^{m} =$$

$$= R\, a_{\overline{m}|i}\,(1+i)^{m} =$$

$$= R\,\frac{1-(1+i)^{-m}}{i}\,(1+i)^{m} =$$

$$= R\,\underbrace{\frac{(1+i)^{m}-1}{i}}_{s_{\overline{m}|i}} = R\, s_{\overline{m}|i}$$

EX14: Calculate the present value of an ordinary annuity of amount \$100 paid annually for 5 years at the rate of interest of 9% . Also calculate its future value at time 5.



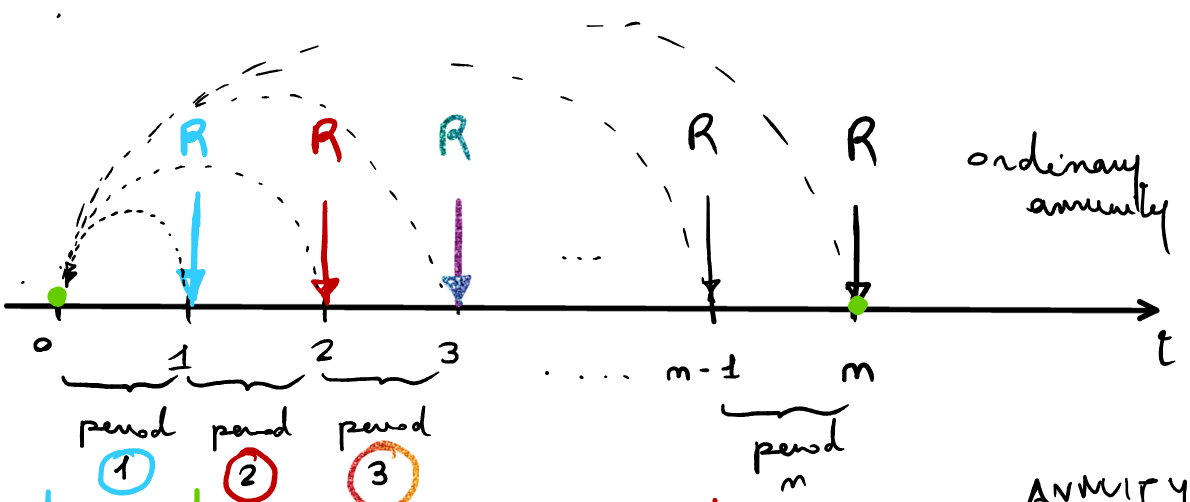
$$w(\underline{R}, 0) = R \, \frac{1 - (1+i)^{-m}}{i}$$

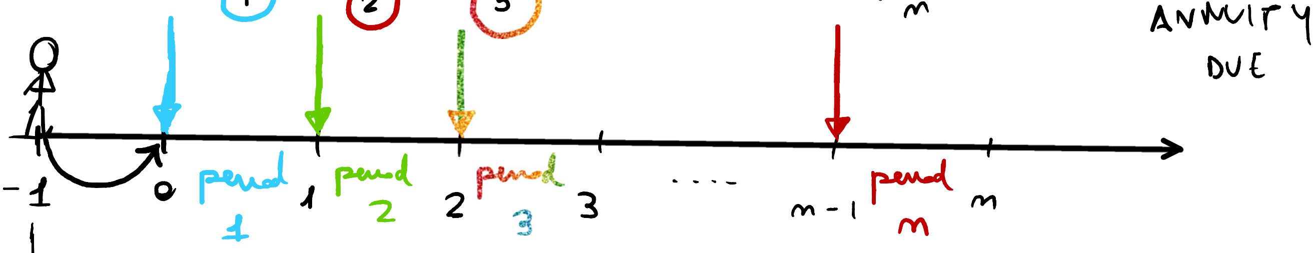
$$w(\underline{100\$}, 0) = 100\$ \cdot \frac{1 - (1+0.09)^{-5}}{0.09} = 388.97\$$$

$$w(\underline{100\$}, 5) = w(\underline{100\$}, 0)\,(1+i)^{m} =$$

$$= 388.97\$\,(1+0.09)^{5} = 598.47\$$$

388.9651





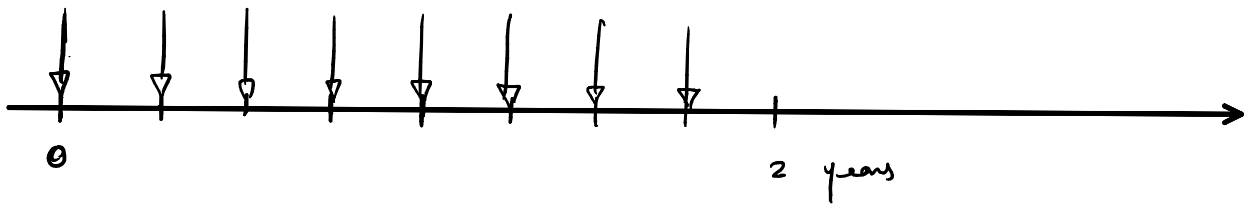
$$w(\underline{R}, -1) = R \, \frac{1 - (1+i)^{-m}}{i} = R \, a_{\overline{m}|i}$$

$$w(\underline{R}, 0) = w(\underline{R}, -1)(1+i)^{1} = w(\underline{R}_1, -1)(1+i) =$$

$$= \underbrace{R\, a_{\overline{m}|i}\,(1+i)}_{\ddot{a}_{\overline{m}|i}}$$

a dots angle m at i

EX15: Find the present value of an annuity-due of 200 euro per quarter (every three-months) for 2 years, if the monthly interest rate is 0.08/12.



$$m = 8$$

$$i_{12} \rightarrow i_4$$

$$\boxed{i_m = (1 + i_m)^{\frac{m}{m}} - 1}$$

$$i_4 = (1 + i_{12})^{\frac{12}{4}} - 1 = \left(1 + \frac{0.08}{12}\right)^3 - 1 = 0.0201$$

$$w(\underline{R}, 0) = R\, a_{\overline{m}|i_4}(1 + i_4) =$$

$$= R\, \frac{1 - (1+i_4)^{-m}}{i_4}(1 + i_4) = 1493.7\ €$$

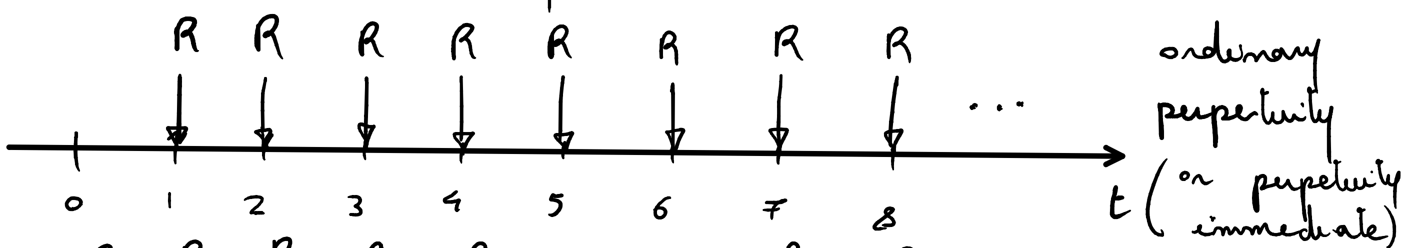
The value after the 2nd period is:

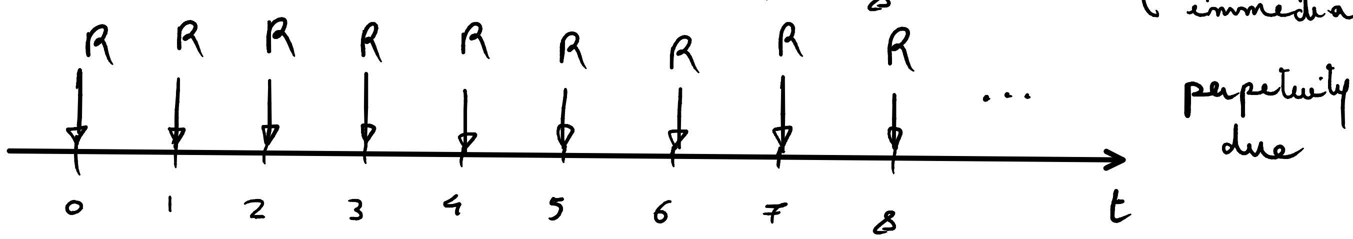
The value after the 2nd period is:

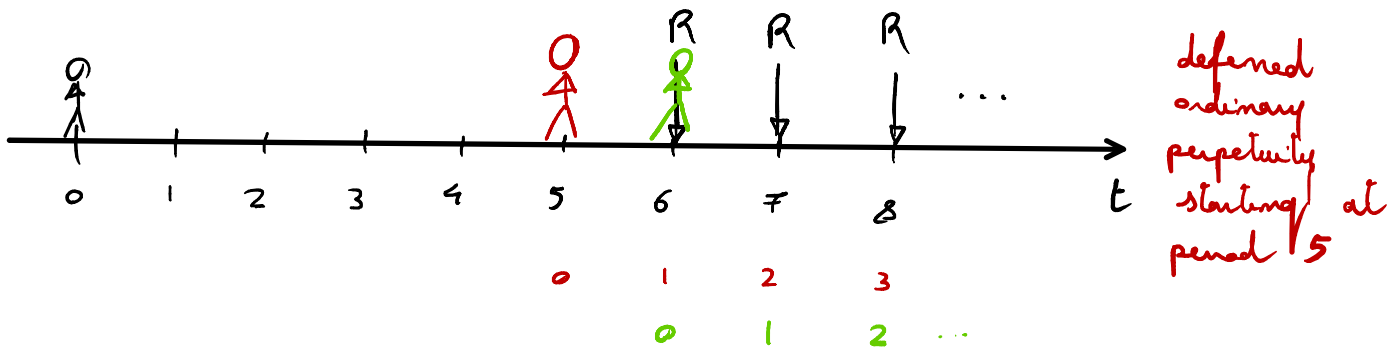
$$w(\underline{R}, 2) = w(\underline{R}, 0)(1 + i_4)^2 =$$

$$= 1493.7\,€\,(1 + 0.0201)^2 = 1554.5\,€$$

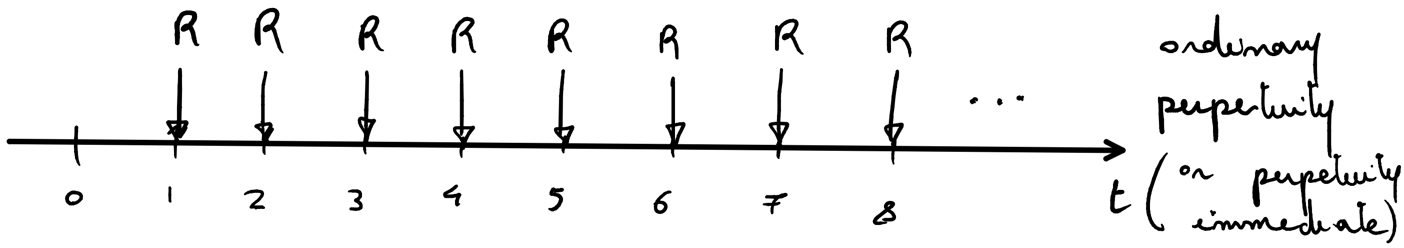
PERPETUITY







deferred perpetuity due starting at period 6



$$w(\underline{R}, 0) = R \, \frac{1 - (1+i)^{-n}}{i} = \text{ordinary annuity}$$

$$= R \, \frac{1 - \frac{1}{(1+i)^{n}}}{i}$$

$$a_n = \left(\frac{1}{2}\right)^n \qquad n \in \mathbb{N} \qquad \text{sequence}$$

n	a_n
0	$\left(\frac{1}{2}\right)^0 = 1$
1	$\left(\frac{1}{2}\right)^1 = \frac{1}{2} = 0.5$
2	$\left(\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25$
3	$\left(\frac{1}{2}\right)^3 = \frac{1}{8} = 0.125$
$\vdots$	
10	$\left(\frac{1}{2}\right)^{10}$

$$\lim_{n \to +\infty} \left(\frac{1}{2}\right)^n = 0$$

because we can get close to 0 as much as we want

So, for an ordinary perpetuity

$$w(\underline{R}, 0) = \lim_{n \to +\infty} R \, \frac{1 - (1+i)^{-n}}{i} =$$

... take 1

$$W(\underline{R},0) = \lim_{n \to +\infty} R \, \frac{\quad}{i} =$$

$$= \lim_{n \to +\infty} R \, \frac{1 - \cancel{\frac{1}{(1+i)^n}}}{i} =$$

something like $\frac{1}{2^n}$

0

$$= R \cdot \frac{1}{i} = R \, a_{\overline{\infty}|i}$$

a angle infinity at i

$a_{\overline{n}|i}$ for annuities

Annuity due:

$$W(\underline{R},0) = R \, \ddot{a}_{\overline{n}|i} = R(1+i) \, \frac{1-(1+i)^{-n}}{i}$$

Perpetuity due:

$$W(\underline{R},0) = \lim_{n \to +\infty} R \, \ddot{a}_{\overline{n}|i} =$$

$$= \lim_{n \to +\infty} R(1+i) \, \frac{1 - \cancel{\left(\frac{1}{1+i}\right)^n}}{i} =$$

0

$$= R \, \frac{1+i}{i} = \underbrace{R\left(1+\frac{1}{i}\right)}_{\ddot{a}_{\overline{\infty}|i}}$$