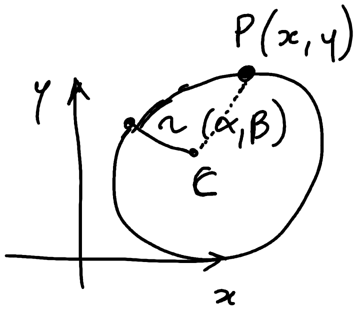
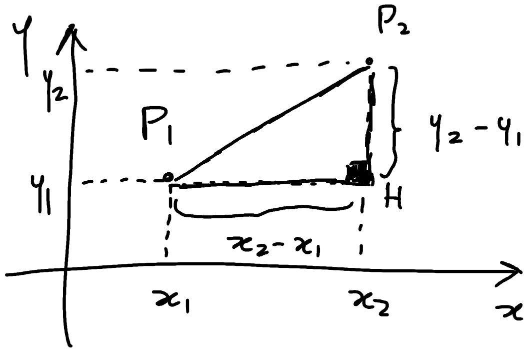
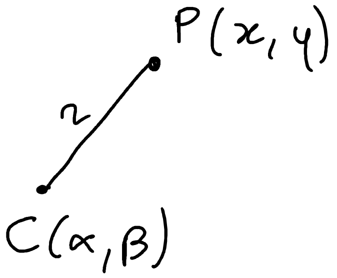
CONIC SECTIONS

circle: $(x-\alpha)^2 + (y-\beta)^2 = r^2$





$$d(P_1, P_2) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$



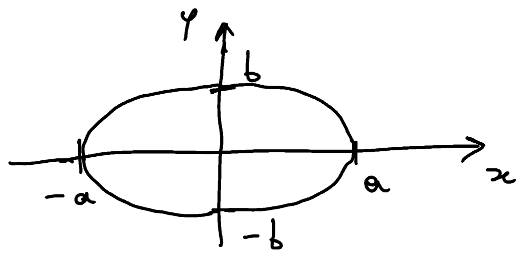
$$d(P, C) = r$$

$$\sqrt{(x-\alpha)^2 + (y-\beta)^2} = r$$

$$(x-\alpha)^2 + (y-\beta)^2 = r^2$$

Ellipse

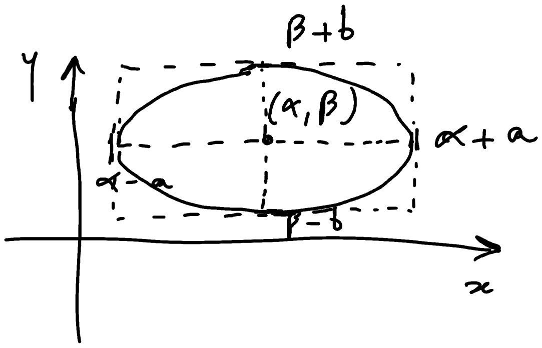
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$\beta+b$

Translated ellipse

$$\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1$$

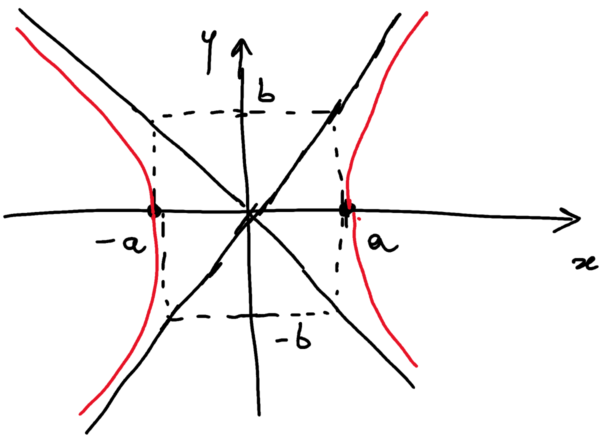


Hyperbola

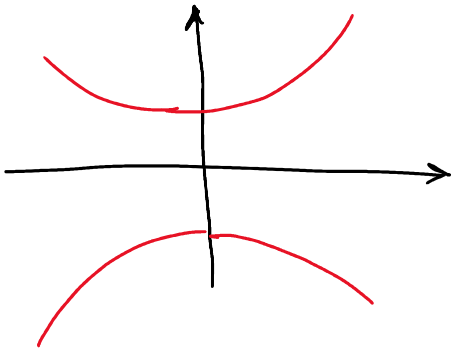
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$y = 0 \qquad \frac{x^2}{a^2} = 1$

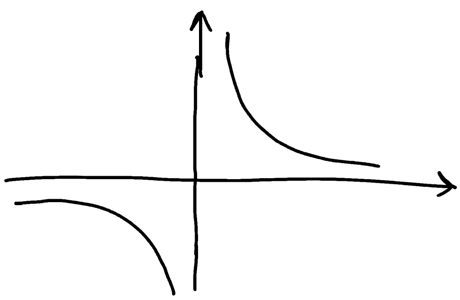
$x = \pm a$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

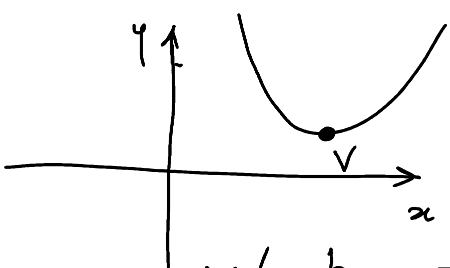


$$xy = k$$



Parabola

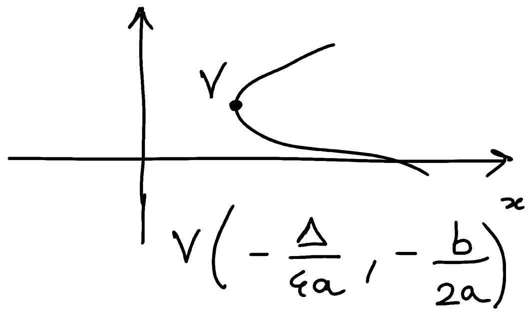
$$y = ax^2 + bx + c$$



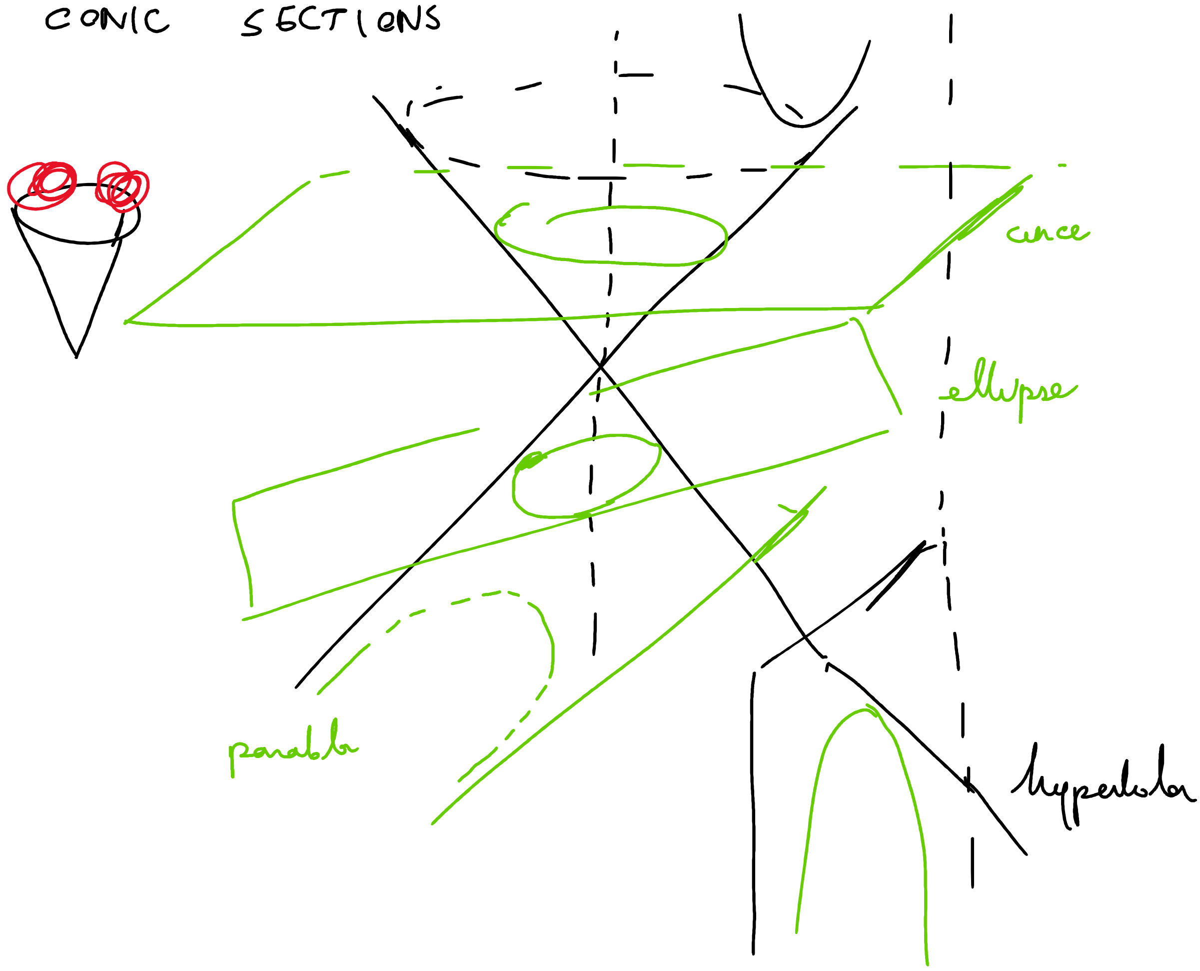
$$V\left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right)$$

$$\Delta = b^2 - 4ac$$

$$x = ay^2 + by + c$$



CONIC SECTIONS



DOMAIN OF FUNCTIONS

$$f(x,y) = \frac{\sqrt[3]{x-y+2}}{x-y} + \ln(9 - x^2 - y^2)$$

$\sqrt{-4} = x \qquad x^2 = -4$ impossible in $\mathbb{R}$

$\sqrt[3]{-8}$ $\qquad$ x^3

$$\sqrt[3]{-8} = x \qquad x^3 = -8 \qquad x = -2$$

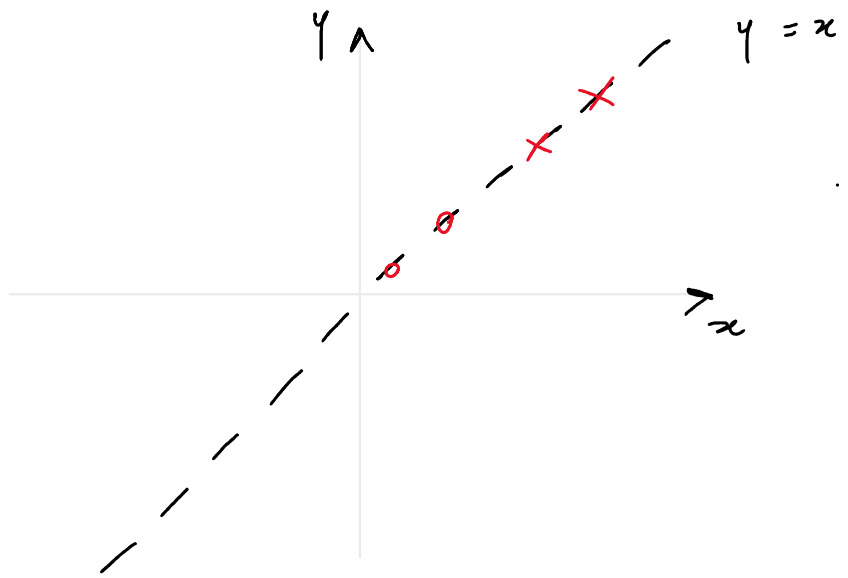
$$(-2)^3 = (-2)\cdot(-2)(-2) = -8$$

$$\ln(\ldots.) \longrightarrow \ldots. > 0$$

$$\begin{cases} x - y \neq 0 \\ 9 - x^2 - y^2 > 0 \end{cases}$$

$x - y \neq 0$

$y \neq x$ ~~evertry~~ everything that is not $y = x$



$$9 - x^2 - y^2 > 0$$

$$9 > x^2 + y^2$$

$$x^2 + y^2 < 9$$

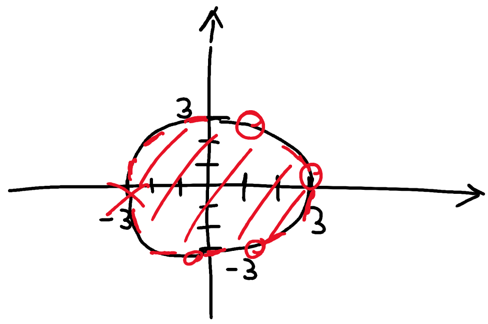
associated equation: $x^2 + y^2 = 9$

associated equation : $x^2 + y^2 = 9$

$$(x-0)^2 + (y-0)^2 = 3^2$$

circle with center $(0,0)$ and radius 3

ray

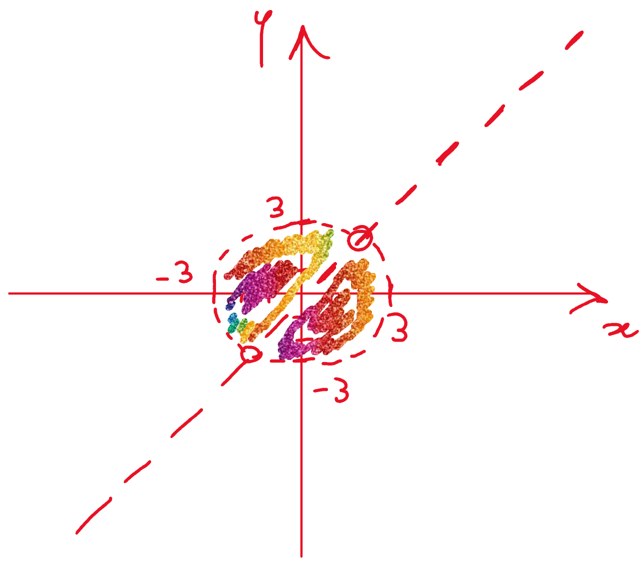


Test point : $(0,0)$

$$x^2 + y^2 < 9$$

$$0^2 + 0^2 < 9$$

$$0 < 9 \quad \text{yes}$$



$$f(x,y) = \sqrt{x^2 - 5x + 6} - \ln(2x - 3y + 6)$$

$$\begin{cases} x^2 - 5x + 6 \geq 0 \\ 2x - 3y + 6 > 0 \end{cases}$$

$\wedge$ and

et

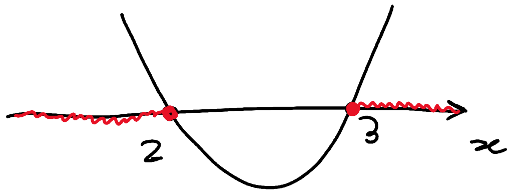
$$x^2 - 5x + 6 \geq 0$$

$$x^2 - 5x + 6 = 0$$

$$x = 2 \quad \vee \quad x = 3$$

or

vel



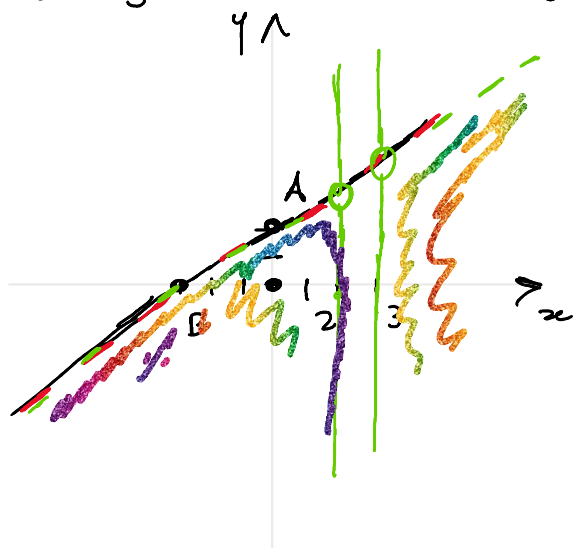
$$\begin{cases} x \leq 2 \quad \vee \quad x \geq 3 \\ 2x - 3y + 6 > 0 \end{cases}$$

$$2x - 3y + 6 = 0$$

$x = 0 \qquad -3y + 6 = 0 \qquad y = 2 \qquad A(0,2)$

$y = 0 \qquad 2x + 6 = 0 \qquad x = -3 \qquad B(-3,0)$

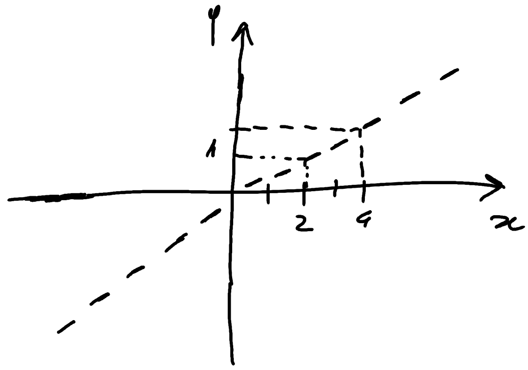
$(0,0) \qquad 2 \cdot 0 - 3 \cdot 0 + 6 > 0 \qquad 6 > 0$ yes



$$f(x,y) = \frac{xy}{\sqrt[3]{x - 2y}} + \frac{\ln(x^2 + y^2 - 4)}{\sqrt{9 - x^2 - y^2}}$$

$$\begin{cases} x - 2y \neq 0 \\ x^2 + y^2 - 4 > 0 \\ 9 - x^2 - y^2 > 0 \end{cases}$$

$x - 2y \neq 0$ $\qquad$ $y \neq \frac{1}{2} x$



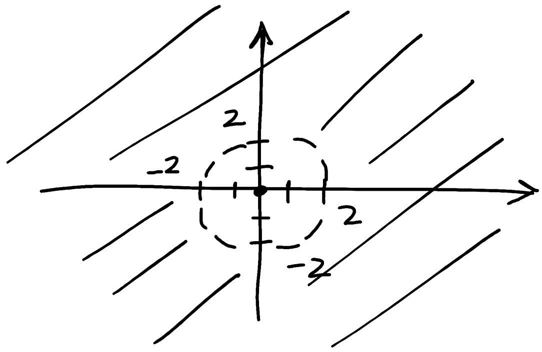
$x^2 + y^2 - 4 > 0$

$x^2 + y^2 > 4$

$x^2 + y^2 = 4$

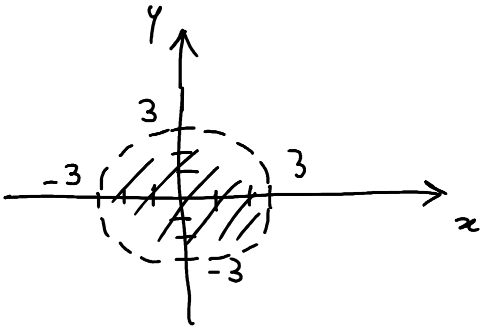
$(x-0)^2 + (y-0)^2 = 2^2$

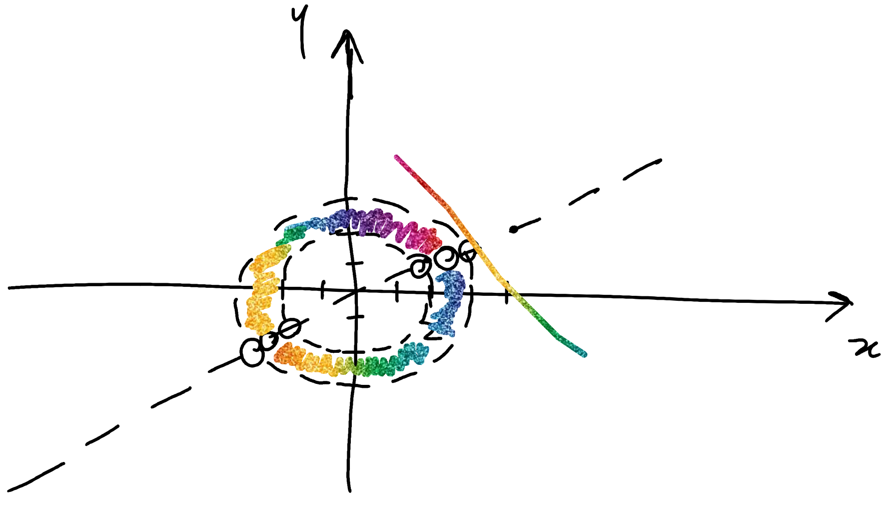
test point $(0,0)$: $0^2 + 0^2 > 4$ $\qquad$ $0 > 4$ $\qquad$ no



$9 - x^2 - y^2 > 0$

$x^2 + y^2 < 9$

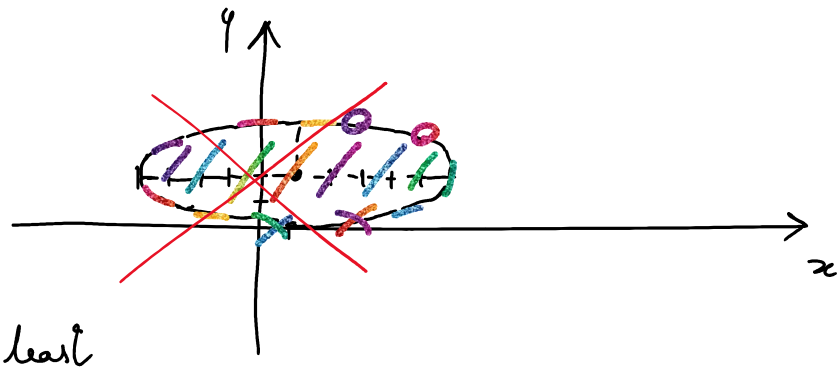




$$f(x,y) = \frac{\ln\left(x^2+y^2-2x-4y-4\right)}{\sqrt[4]{1 - \frac{(x-1)^2}{25} - (y-2)^2}}$$

$$\begin{cases} x^2 + y^2 - 2x - 4y - 4 > 0 \\ 1 - \frac{(x-1)^2}{25} - (y-2)^2 > 0 \end{cases}$$

$$\frac{(x-1)^2}{5^2} + \frac{(y-2)^2}{1^2} < 1 \qquad C\,(1,2)$$



At least

Two options for the test point:

1) the origin: $\frac{1}{25} + 4 < 1$ NO

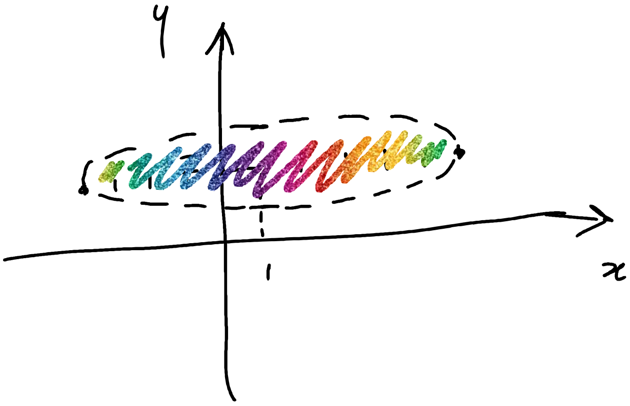
2) the center the ellipse: $(1,2)$

$$\frac{(x-1)^2}{5^2} + \frac{(y-2)^2}{1^2} < 1$$

$$\frac{(1-1)^2}{5^2} + \frac{(2-2)^2}{1^2} < 1$$

$$0 < 1 \qquad \text{yes}$$

$0 < 1$ yes



$$x^2 + y^2 - 2x - 4y - 4 > 0$$

$$x^2 + y^2 - 2x - 4y - 4 = 0$$

complete the square

$$(A + B)^2 = A^2 + 2AB + B^2$$

1 rearrange

$$x^2 - 2x + y^2 - 4y = 4$$

$$x^2 + 2 \cdot x \cdot (-1) + y^2 + 2 \cdot y\,(-2) = 4$$

$$A^2 + 2 \cdot A \cdot B$$

$$x^2 + 2x \cdot (-1) + (-1)^2 + y^2 + 2 \cdot y\,(-2) + (-2)^2 = 4 + (-1)^2 + (-2)^2$$

$$x^2 - 2x + 1 + y^2 - 4y + 4 = 9$$

$$(x-1)^2 + (y-2)^2 = 3^2$$

or

$$x^2 + y^2 + ax + by + c = 0$$

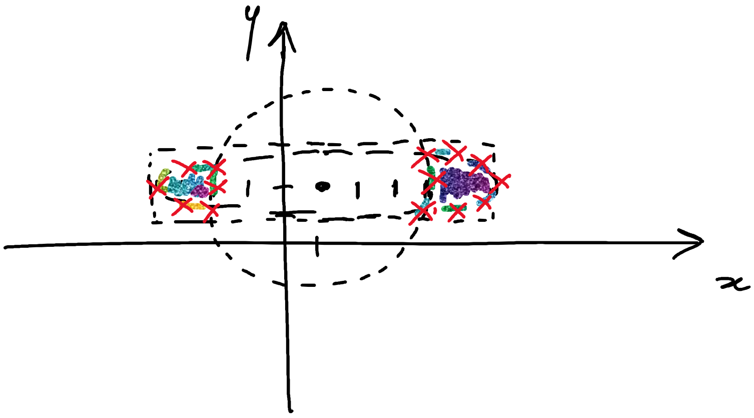
$$x^2 + y^2 + ax + by + c = 0$$

$$\alpha = -\frac{a}{2} \qquad \beta = -\frac{b}{2}$$

$$r = \sqrt{\underbrace{\left(-\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2 - c}_{\geq 0}}$$

$$(x-1)^2 + (y-2)^2 = 3^2$$

$$(x-1)^2 + (y-2)^2 > 3^2$$



$$x^2 + y^2 - 2x - 4y - 4 = 0$$

$$a = -2 \qquad b = -4 \qquad c = -4$$

$$\alpha = -\frac{a}{2} = 1 \qquad \beta = -\frac{b}{2} = 2$$

$$\left(-\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2 - c = 1^2 + 2^2 + 4 = 9 \geq 0$$

$$r = \sqrt{9} = 3$$