(1) $x_3 = x_1^2 + x_2^2$

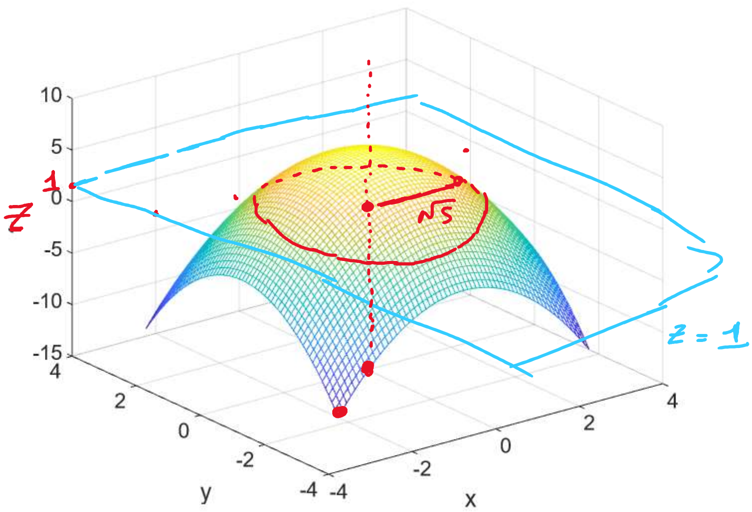
(2) $x_3 = -x_1^2 - x_2^2$

(3) $x_3 = x_1^2 - x_2^2$

(4) $x_3 = (x_1 + x_2)^2$

(5) $x_3 = -(x_1 + x_2)^2$

$$z = 6 - x^2 - y^2$$



Consider the equation $z = 1$

This means: take all those point having $z = 1$. For example:

$(0, 0, 1)$; $(1, 0, 1)$; $(-2, 1, 1)$

So it is a plane $\parallel$ // to the xy axes

$\parallel$ or // parallel

$\perp$ perpendicular (or orthogonal)

The intersection between our surface and the plane $z = 1$ is a circle consisting in all those points whose "height" is equal to 1.

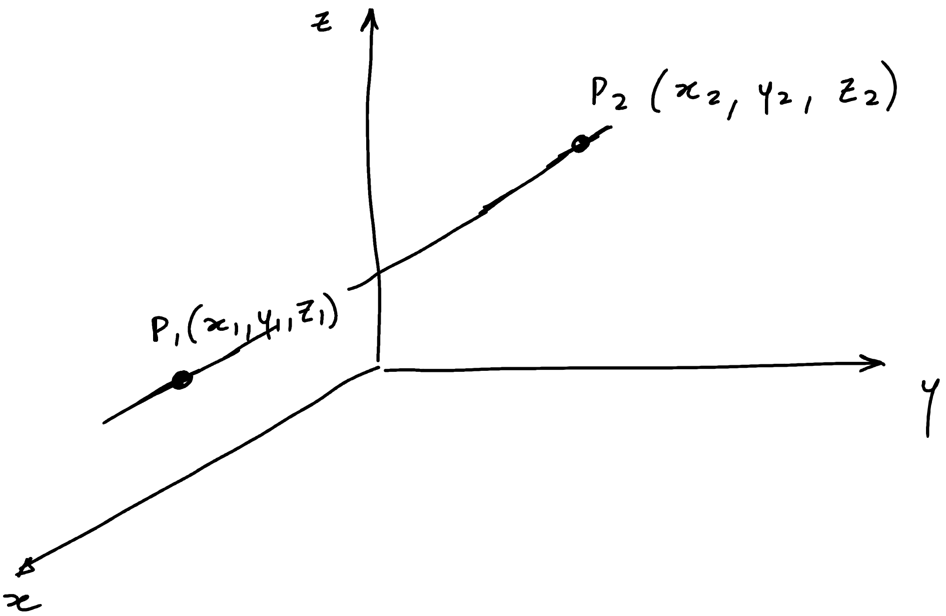
those points whose "height" is equal to 1.
This is the concept of level curves.
This concept can be generalized to a generic level K. In this case the equation of the plane is $z = K$.

$$\begin{cases} z = 6 - x^2 - y^2 \\ z = 1 \end{cases}$$

$$1 = 6 - x^2 - y^2$$

$x^2 + y^2 = 5$ equation of a circle with centre $(0,0)$, $r = \sqrt{5}$.

$(x - \alpha)^2 + (y - \beta)^2 = r^2$ centre (α, β), radius r



myArray = [1, 2, 5, 7];

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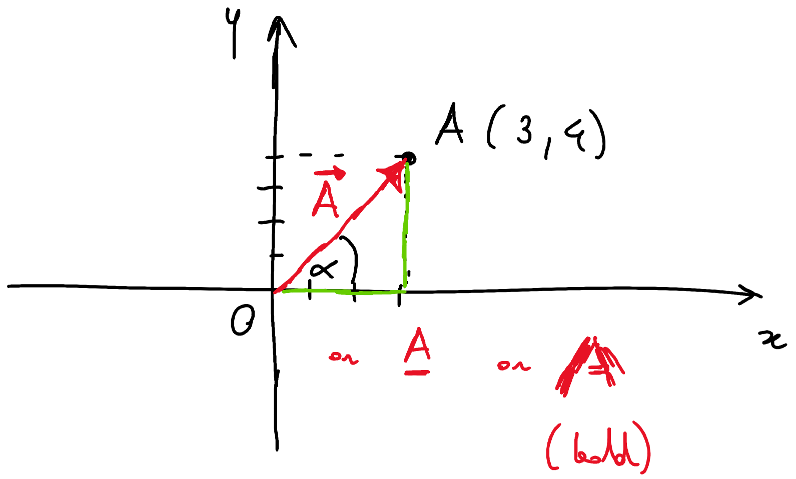
A vector is an entity characterized by:

1) magnitude (or modulus)
2) direction
3) verse



$A = [3 \;\; 4]$

1) Magnitude: it is the length of the arrow

$$A = \sqrt{3^2 + 4^2} = 5$$

In general if $\vec{A} = (A_x, A_y)$

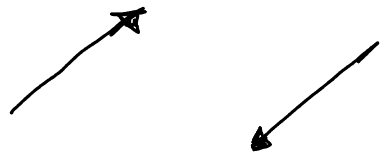
components

The magnitude is $A = \sqrt{A_x^2 + A_y^2}$

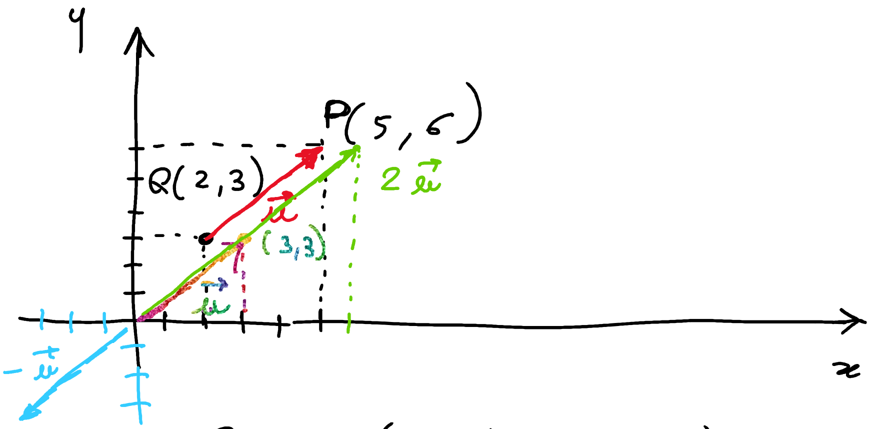
2) Direction: is the angle that the vector forms with the x axis

3) Verse: the orientation of the arrow

3) Verse: the orientation of the arrow



same magnitude, same direction, opposite verse



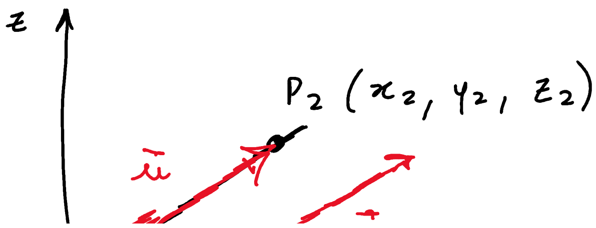
$\vec{u} = P - Q = (5,6) - (2,3) = (3,3)$

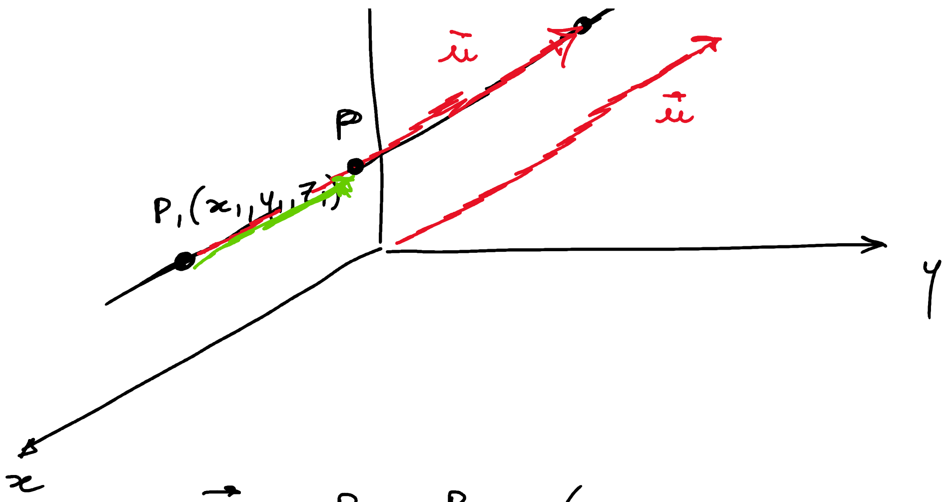
$2\vec{u} = (6,6)$ in green

$-\vec{u} = (-3,3)$

$-\vec{u}$ and $2\vec{u}$ are $\parallel$ (parallel) to the original red vector $\vec{u} = P - Q$.

In general, multiply a vector by a scalar, yields a parallel vector.





$$\vec{u} = P_2 - P_1 = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

Consider a generic point on the line $P(x, y, z)$ (not known)

The green vector is $P - P_1 = (x - x_1, y - y_1, z - z_1)$ and is parallel to $\vec{u}$, so there exists a scalar $t \in \mathbb{R}$: (: or | means such that)

$$P - P_1 = t\,\vec{u}$$

$$(x - x_1, y - y_1, z - z_1) = t\,(x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

$$(t(x_2 - x_1), t(y_2 - y_1), t(z_2 - z_1))$$

$$\begin{cases} x - x_1 = t\,(x_2 - x_1) \\ y - y_1 = t\,(y_2 - y_1) \\ z - z_1 = t\,(z_2 - z_1) \end{cases}$$

parametric equation of the line in the space

In the plane

$$
\begin{cases}
x - x_1 = t(x_2 - x_1) \\
y - y_1 = t(y_2 - y_1) \\
\cancel{z - z_1 = t(z_2 - z_1)}
\end{cases}
\qquad
\begin{cases}
t = \dfrac{x - x_1}{x_2 - x_1} \\[2ex]
t = \dfrac{y - y_1}{y_2 - y_1}
\end{cases}
$$

suppose $x_1 \neq x_2$ and $y_2 \neq y_1$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Plot the level curves of:

$$z = \frac{x}{y}$$

$$
\begin{cases}
z = \dfrac{x}{y} \\
z = k
\end{cases}
\qquad \frac{x}{y} = k \quad \text{for some } k \in \mathbb{R}
$$

If $y \neq 0$ (and it is indeed $y \neq 0$ because of the domain of the function)

$$x = yk$$

$$y = \frac{1}{k}x \qquad \text{line passing through the origin}$$