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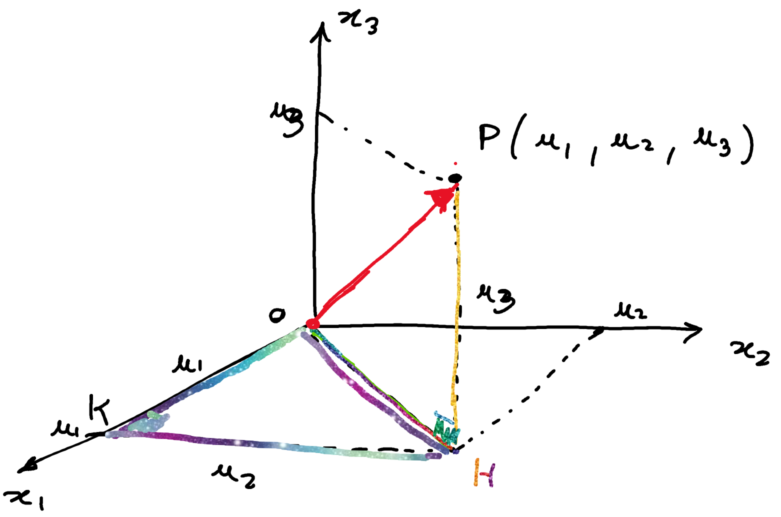
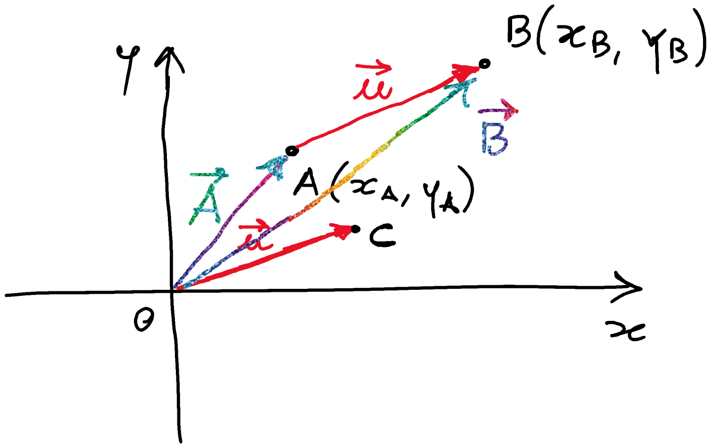


$$d(P, O) = \|\vec{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

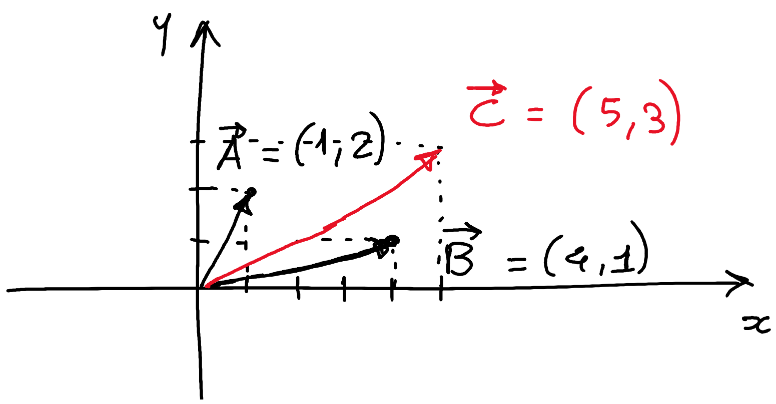


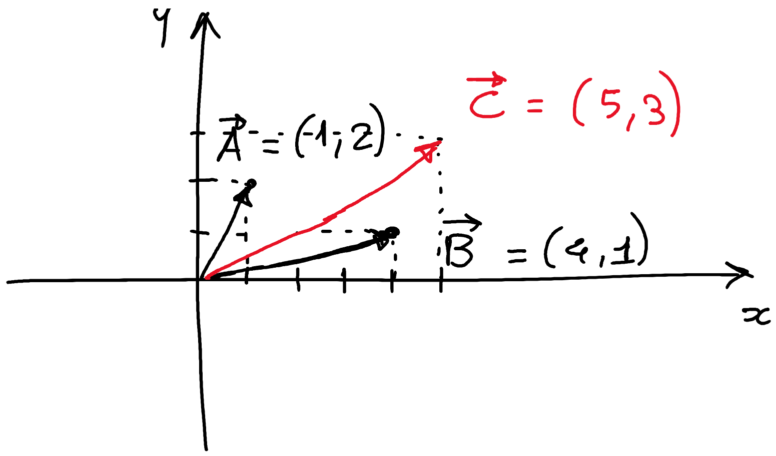
$$\vec{u} = B - A =$$

$$= (x_B - x_A, y_B - y_A)$$

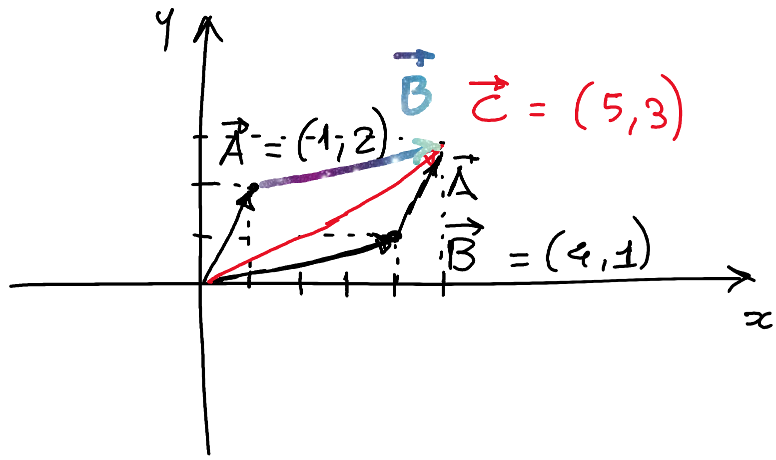
$$d(A, B) = \|\vec{u}\|$$

$$\vec{u} = \vec{B} - \vec{A}$$

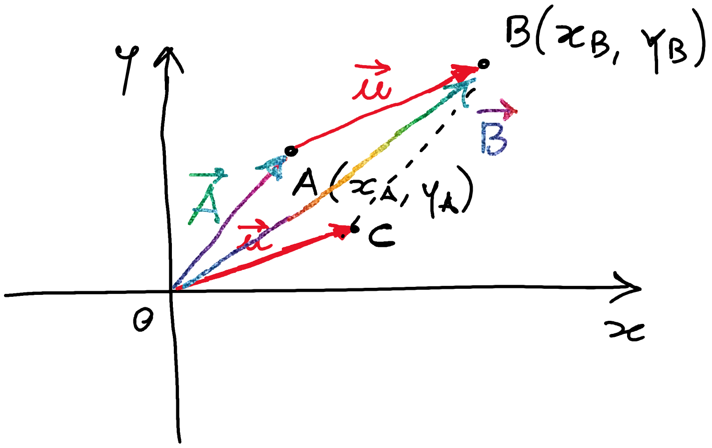




$$\vec{A} + \vec{B} = (1,2) + (4,1) = (1+4,\ 2+1) = (5,3)$$



parallelogram law



$$\vec{A} + \vec{u} = \vec{B}$$

$$\vec{u} = \vec{B} - \vec{A} = (x_B, y_B) - (x_A, y_A) = (x_B - x_A, y_B - y_A)$$

$$\vec{u} = B - A = (x_B, y_B) - (x_A, y_A) = (x_B - x_A, y_B - y_A)$$

$$d(A, B) = \|\vec{u}\| = \|\vec{B} - \vec{A}\| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

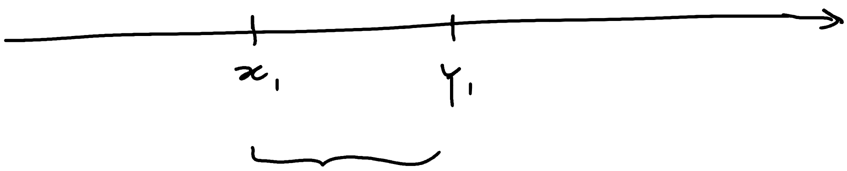
Def: Distance between two vectors

Let $\underline{x} = (x_1, \ldots, x_n) \in \mathrm{R}^n$, $\underline{y} = (y_1, \ldots, y_n) \in \mathrm{R}^n$.

The DISTANCE between $\underline{x}$ and $\underline{y}$ is the following not negative number:

$$d(\underline{x}, \underline{y}) = \|\underline{x} - \underline{y}\| = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

In one dimension



$$d = y_1 - x_1$$

$$d = x_1 - y_1$$

In general: $d = |x_1 - y_1| = |y_1 - x_1|$

Def: Distance between two vectors

Let $\underline{x} = (x_1, \ldots, x_n) \in \mathrm{R}^n$, $\underline{y} = (y_1, \ldots, y_n) \in \mathrm{R}^n$.

The DISTANCE between $\underline{x}$ and $\underline{y}$ is the following not negative number:

$$d(\underline{x}, \underline{y}) = \|\underline{x} - \underline{y}\| = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

If $n = 1$

$$\sqrt{(x_1 - y_1)^2} = |x_1 - y_1|$$

$$\infty$$

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

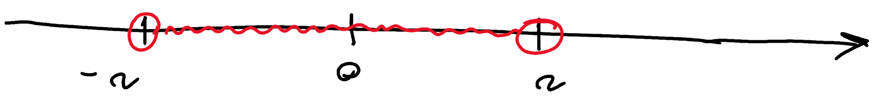
Known

$$|x| < r \qquad r \geq 0$$

$$x < r \quad \text{if } x \geq 0$$
or
$$-x < r \quad \text{if } x < 0$$

$$\infty$$

$$x < r \quad \text{if } x \geq 0$$
$$x > -r \quad \text{if } x < 0$$



$$-r < x < r$$

Saying $|x| < r$ is equivalent to $-r < x < r$

$$\infty$$

? Known Known

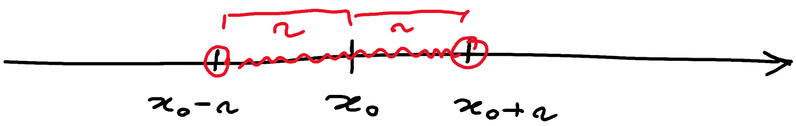
? known known

$$|x - x_0| < r$$

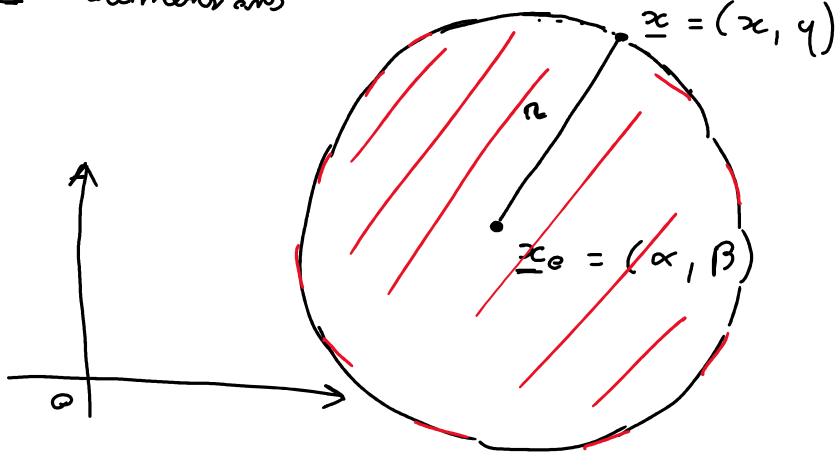
$$-r < x - x_0 < r$$

$$-r + x_0 < x - \cancel{x_0} + \cancel{x_0} < r + x_0$$

$$x_0 - r < x < x_0 + r$$



2 dimensions



$$d(\underline{x}, \underline{x_0}) = r \qquad\qquad d(\underline{x}, \underline{x_0}) < r$$

$$\| \underline{x} - \underline{x_0} \| = r$$

etc.

$$\sqrt{(x-\alpha)^2 + (y-\beta)^2} = r$$

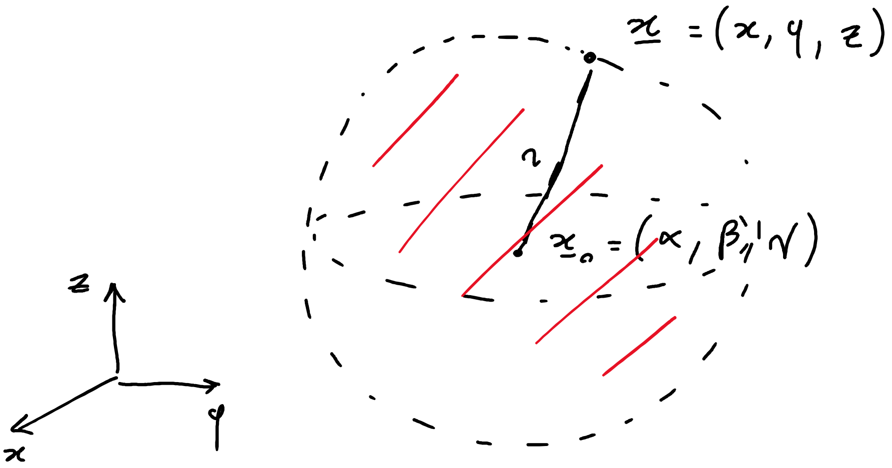
$$(x-\alpha)^2 + (y-\beta)^2 = r^2$$

$(x - \alpha)^2$

$$(x-\alpha)^2+(y-\beta)^2=r^2 \qquad\qquad (x-\alpha)^2+(y-\beta)^2<r^2$$

3 dimensions. In a similar way, we get:

$$d(\underline{x},\underline{x}_0)=r$$



$$d(\underline{x},\underline{x}_0)<r$$

$$(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2=r^2$$

$<$

Def: Neighborhood of $\underline{x}_0$

Let $\underline{x}\in R^n$ and $r\in R, r>0$

A NEIGHBORHOOD of $\underline{x}_0$ with radius r is given by:

$$B(\underline{x}_0,r)=\{\underline{x}\in R^n : d(\underline{x}_0,\underline{x})<r\}$$

Unconstrained optimization

Def: absolute (or global) maximum point and absolute (or global) minimum point

Let $f : A \subseteq R^n \to R$ and $\underline{x}^* \in A$

or global

$\underline{x}^*$ is an ABSOLUTE MAXIMUM (MAX) point if

$$f(\underline{x}^*) \geq f(\underline{x}) \qquad \forall \underline{x} \in A$$

or global

$\underline{x}^*$ is an ABSOLUTE MINIMUM (MIN) point if

$$f(\underline{x}^*) \leq f(\underline{x}) \qquad \forall \underline{x} \in A$$

Notice: If a point is an absolute max then there are no points in the domain at which f takes a larger value

A denotes the domain of the function f

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