Def: relative (or local) maximum point and relative (or local) minimum point

Let $f : A \subseteq R^n \to R$ and $\underline{x}^* \in A$

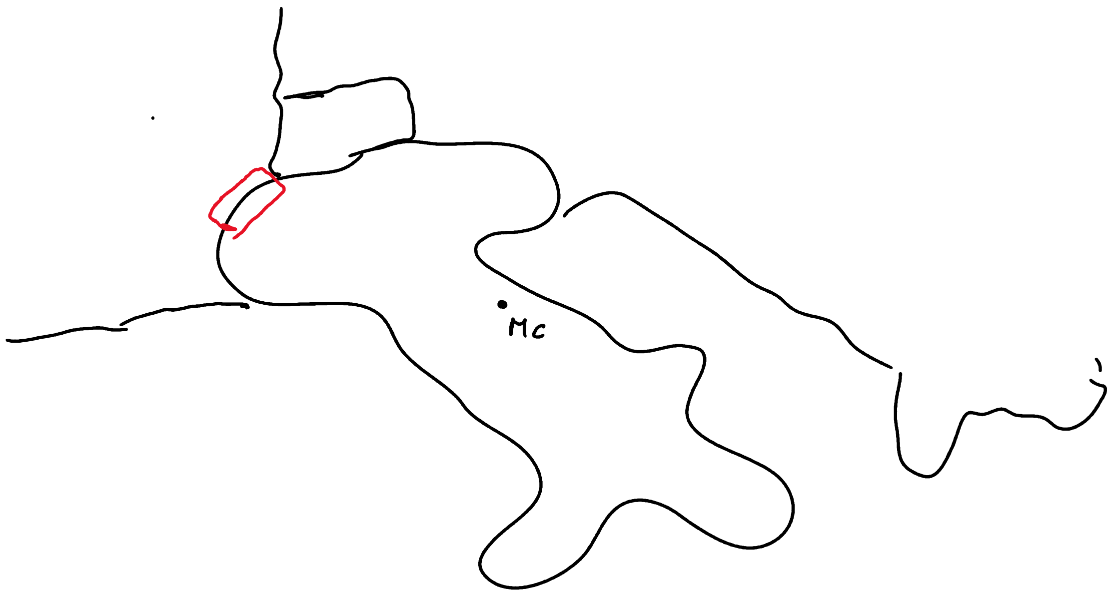
$\underline{x}^*$ is a RELATIVE MAXIMUM point if

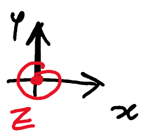
$$\exists\, B(\underline{x}^*, r):\ f(\underline{x}^*) \geq f(\underline{x}) \quad \forall \underline{x} \in B(\underline{x}^*, r) \cap A$$

$\underline{x}^*$ is an RELATIVE MINIMUM point if

$$\exists\, B(\underline{x}^*, r):\ f(\underline{x}^*) \leq f(\underline{x}) \quad \forall \underline{x} \in B(\underline{x}^*, r) \cap A$$

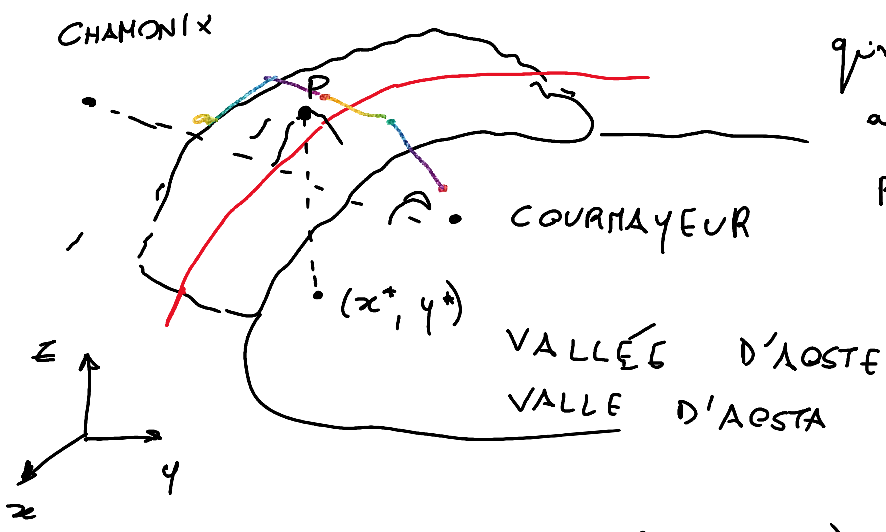
Notice: If a point is a local max then there are no nearby points at which f takes a larger value





$z = f(x, y)$
gives you the

gives you the altitude of point (x,y)



$$P(x^*, y^*, f(x^*, y^*))$$

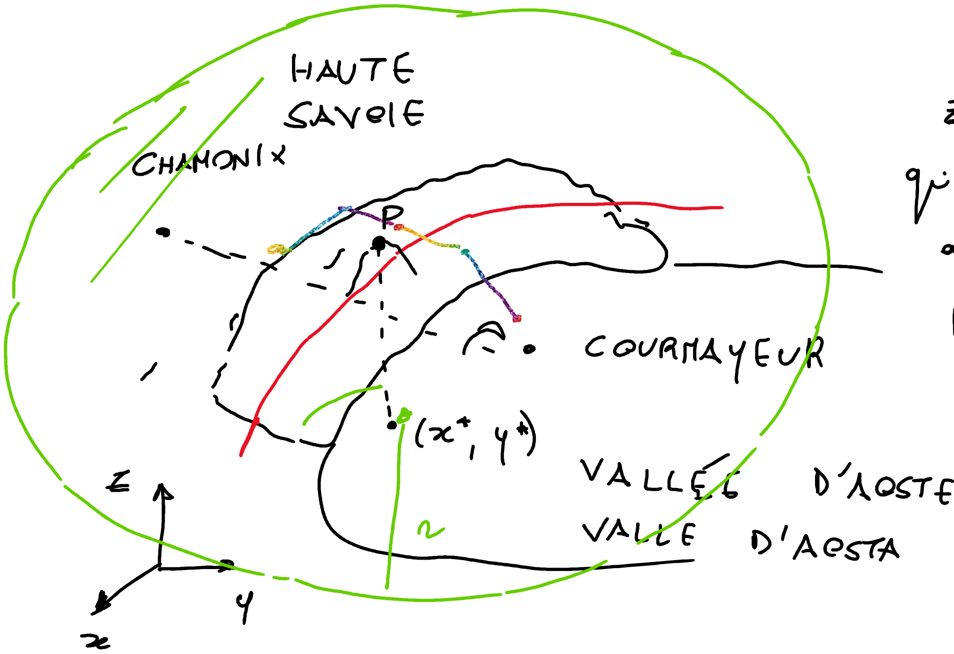
A: domain of our function

In this example

A : is : $\{(x,y) \in \mathbb{R}^2 : (x,y)$ is in Europe$\}$

$$f : B \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$$

$B = \{(x,y) \in \mathbb{R}^2 : (x,y)$ is on s' in the world$\}$



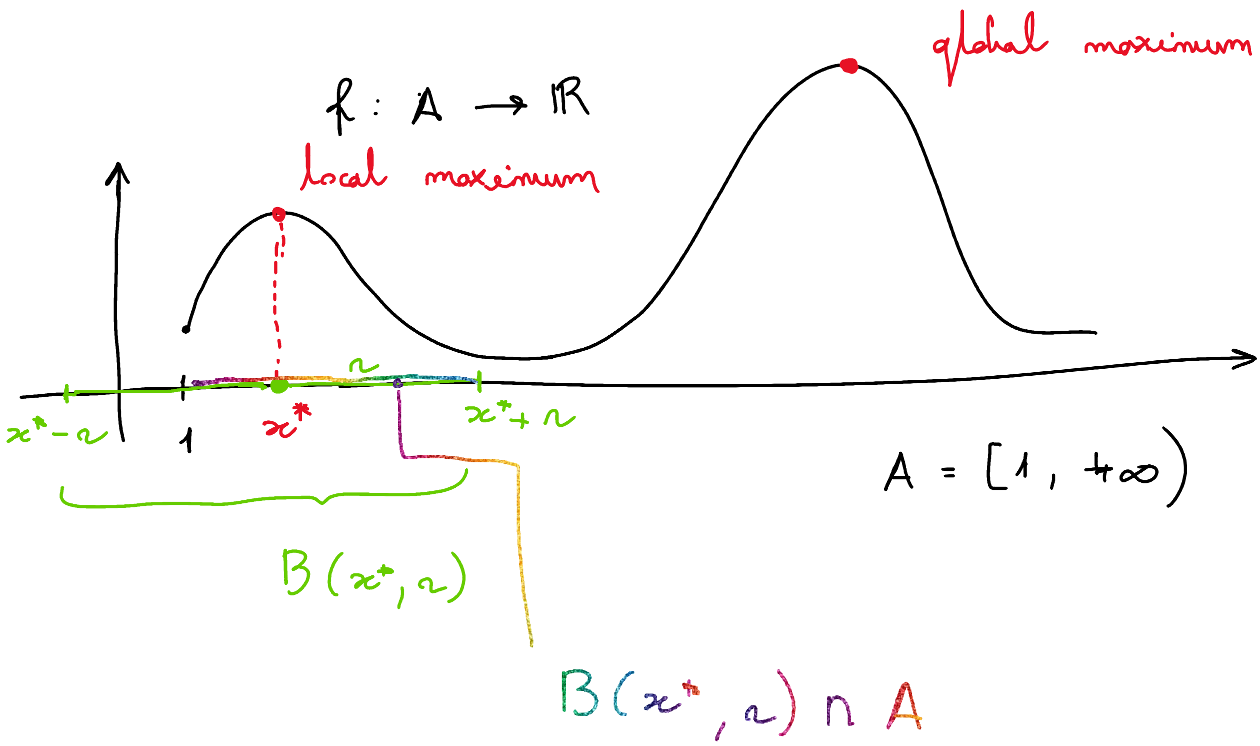
z

gi

al

po

global maximum



For any point x in $B(x^*, r) \cap A$ we have:

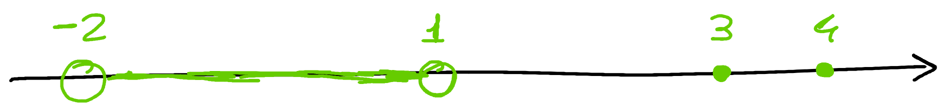
$f(x^*) \geq f(x)$

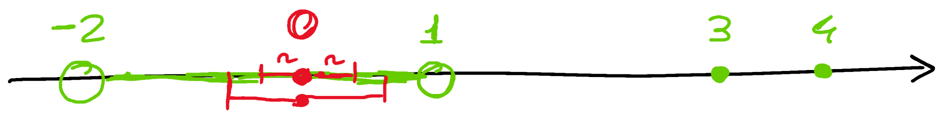
Def: LIMIT of a function

1)A point $\underline{x} \in R^n$ is an **accumulation point** of $A \subseteq R^n$
if in all r - balls of $\underline{x}$ there exists a point of A different from $\underline{x}$

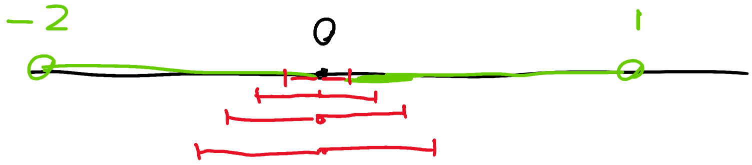
$n = 1$

$A = (-2, 1) \cup \{3, 4\}$

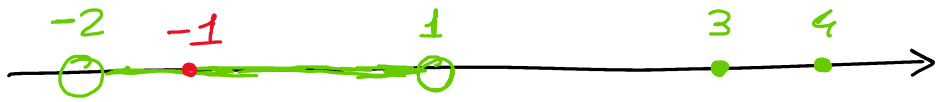




ZOOM

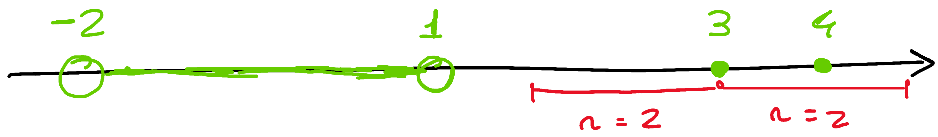


Independently on the value of r, I can always find a point different than 0 lying in the neighborhood. Consequently, 0 is an accumulation point of our set A.

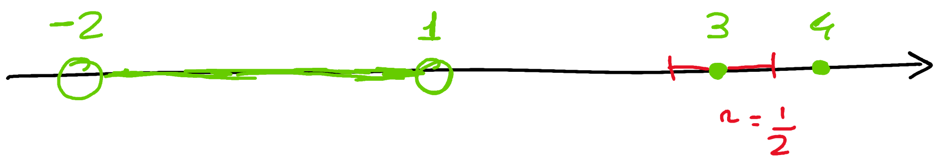


Similarly, -1 is an accumulation point of our set A.

This is also true for any point in the interval (-2, 1).



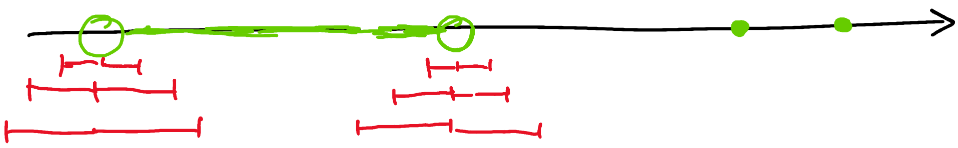
If $r = 2$ there is a point different than 3 that is in the neighborhood



But if $r = \frac{1}{2}$ there is non

Consequently 3 is not an accumulation point of A. Nor is 4.

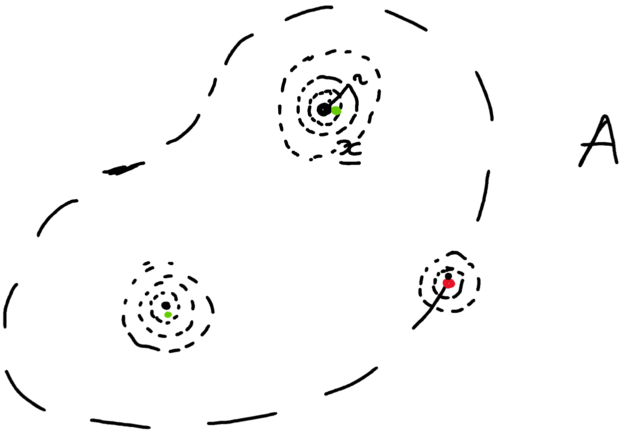
-2 1 3 4



-2 and 1 are accumulation points

The set of all the accumulation points of a set is called the derived set. Fundamentally it is the set of points for which we can compute the limit of a function.

$n = 2$



$\underline{x}$ is an accumulation point

In this example any interior point of A is an accumulation point

But also all the points in the border are accumulation points. Thus, I can compute the limit in any point of A plus its border.

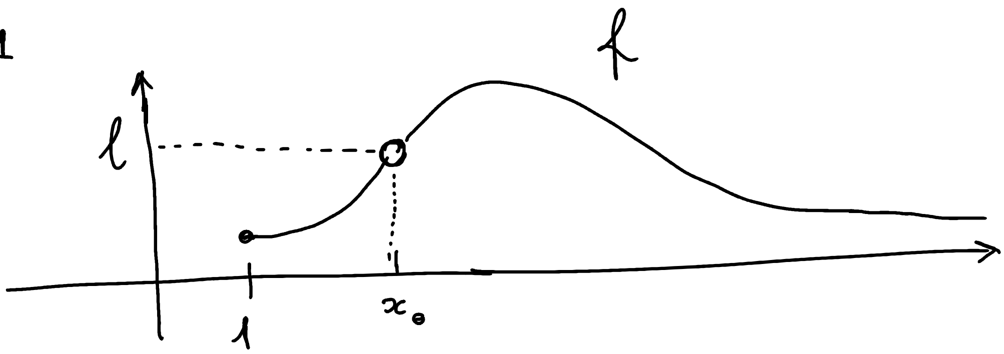
2) Let $f: A \subseteq R^n \to R$ and let $\underline{x}^* = (x_1^*, \dots, x_n^*) \in A$ be an accumulation point of A. Then

$$\lim_{\underline{x} \to \underline{x}^*} f(x_1, \dots, x_n) = l$$

if $\forall \varepsilon > 0 \, \exists \delta > 0$ such that $|f(\underline{x}) - l| < \varepsilon$

$$\forall \underline{x} \in B(\underline{x}^*, \delta) \cap A - \{\underline{x}^*\}$$

$n = 1$



$$A = [1, x_0) \cup (x_0, +\infty)$$

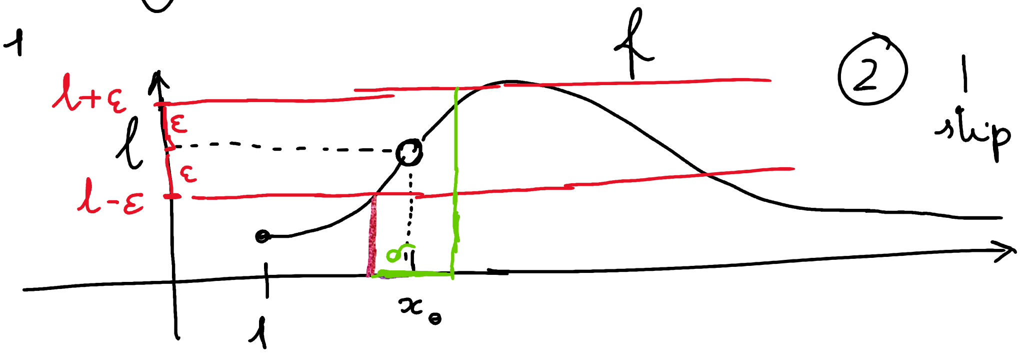
$$\lim_{x \to x_0} f(x) = l$$

$$|f(x) - l| < \varepsilon$$

$$-\varepsilon < f(x) - l < \varepsilon$$

$$l - \varepsilon < f(x) < l + \varepsilon$$ ε is chosen

(1) I choose a value of ε



(2) I build the strip

(3) I can find a δ such that for any point in the neighborhood with center x_0 and ray δ excluding x_0 (and intersected with the domain), the function is trapped within the strip.

If this is not the case, the limit does not exist.