Esercitazione 4a: Metodo del Simplesso

Esercizio 1

Risolvere il seguente modello di Programmazione Lineare attraverso il metodo del Simplesso sia in forma algebrica che tabellare:

$$\max z = 2x_1 + 4x_2 + 3x_3$$

soggetto ai seguenti vincoli:

$$\begin{gathered} 3x_1 + 4x_2 + 2x_3 \leq 60 \\ 2x_1 + x_2 + 2x_3 \leq 40 \\ x_1 + 3x_2 + 2x_3 \leq 80 \\ x_1,\ x_2, x_3 \geq 0. \end{gathered}$$

Esercizio 2

Risolvere il seguente modello di Programmazione Lineare attraverso il metodo del Simplesso sia in forma algebrica che tabellare:

$$\max z = 4x_1 + 3x_2 + 6x_3$$

soggetto ai seguenti vincoli:

$$\begin{gathered} 3x_1 + x_2 + 3x_3 \leq 30 \\ 2x_1 + 2x_2 + 3x_3 \leq 40 \\ x_1,\ x_2, x_3 \geq 0. \end{gathered}$$

Esercizio 3

Risolvere il seguente modello di Programmazione Lineare attraverso il metodo del Simplesso in forma geometrica, algebrica e tabellare:

$$\max z = 4500x_1 + 4500x_2$$

soggetto ai seguenti vincoli:

$$\begin{gathered} x_1 \leq 1 \\ x_2 \leq 1 \\ 5000x_1 + 4000x_2 \leq 6000 \\ 400x_1 + 500x_2 \leq 600 \\ x_1 \geq 0, x_2 \geq 0. \end{gathered}$$

max $\quad z = 2x_1 + 4x_2 + 3x_3$

s. t. $\quad 3x_1 + 4x_2 + 2x_3 \leq 60$

$\quad 2x_1 + x_2 + 2x_3 \leq 40$

$\quad x_1 + 3x_2 + 2x_3 \leq 80$

$\quad x_1, x_2, x_3 \geq 0$

FORMA ALGEBRICA

$$
\begin{array}{lrcl}
 & z - 2x_1 - 4\underline{x_2} - 3x_3 & & = 0 \\
s_1 & 3x_1 + 4x_2 + 2x_3 + \underline{s_1} & & = 60 \\
s_2 & 2x_1 + x_2 + 2x_3 & + \underline{s_2} & = 40 \\
s_3 & x_1 + 3x_2 + 2x_3 & + \underline{s_3} & = 80
\end{array}
$$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Base: $\underline{x}_B = (s_1, s_2, s_3)^T = (60, 40, 80)^T$

Entra in base x_2. Se $x_2 = \theta \geq 0$, abbiamo quanto segue:

$$
\begin{array}{l}
3x_1 + 4\theta + 2x_3 + s_1 = 60 \\
2x_1 + \theta + 2x_3 + s_2 = 40 \\
x_1 + 3\theta + 2x_3 + s_3 = 80
\end{array}
$$

Ma x_1 ed x_3 non sono in base, quindi $x_1 = x_3 = 0$

$$
\begin{array}{l}
4\theta + s_1 = 60 \\
\theta + s_2 = 40 \\
3\theta + s_3 = 80
\end{array}
\quad \Bigg\| \quad
\begin{array}{l}
s_1 = 60 - 4\theta \geq 0 \\
s_2 = 40 - \theta \geq 0 \\
s_3 = 80 - 3\theta \geq 0
\end{array}
\quad \Bigg\| \quad
\begin{cases}
60 - 4\theta \geq 0 \\
40 - \theta \geq 0 \\
80 - 3\theta \geq 0
\end{cases}
$$

$$\begin{cases} 4\theta \leq 60 \\ \theta \leq 40 \\ 3\theta \leq 80 \end{cases} \qquad \begin{cases} \theta \leq 15 \\ \theta \leq 40 \\ \theta \leq \frac{80}{3} \end{cases}$$

Quindi il massimo valore che può assumere θ è 15. Tale valore azzera s_1 che esce dalla base. La nuova base è quindi $(x_2, s_2, s_3)^T$. Per trovare i valori di queste variabili occorre risolvere il sistema in termini di queste variabili.

$$\begin{array}{llr}
 & z - 2x_1 - 4x_2 - 3x_3 = 0 & (R0) \\
 & 3x_1 + 4x_2 + 2x_3 + s_1 = 60 & (R1) \\
s_2 & 2x_1 + x_2 + 2x_3 + s_2 = 40 & (R2) \\
s_3 & x_1 + 3x_2 + 2x_3 + s_3 = 80 & (R3)
\end{array}$$

$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

$$R1 := \frac{1}{4} R_1$$

$$\begin{array}{llr}
 & z - 2x_1 - 4x_2 - 3x_3 = 0 & (R0) \\
 & \frac{3}{4}x_1 + x_2 + \frac{1}{2}x_3 + \frac{1}{4}s_1 = 15 & (R1) \\
s_2 & 2x_1 + x_2 + 2x_3 + s_2 = 40 & (R2) \\
s_3 & x_1 + 3x_2 + 2x_3 + s_3 = 80 & (R3)
\end{array}$$

$$R0 := R0 + 4R1 \; ; \; R2 := R2 - R1 \; ; \; R3 := R3 - 3R1$$

$$\begin{array}{llr}
 & z + x_1 - x_3 + s_1 = 60 & (R0) \\
x_2 & \frac{3}{4}x_1 + x_2 + \frac{1}{2}x_3 + \frac{1}{4}s_1 = 15 & (R1) \\
s_2 & \frac{5}{4}x_1 + \frac{3}{2}x_3 - \frac{1}{4}s_1 + s_2 = 25 & (R2) \\
s_3 & -\frac{5}{4}x_1 + \frac{1}{2}x_3 - \frac{3}{4}s_1 + s_3 = 35 & (R3)
\end{array}$$

Quindi $\underline{x}_B = (x_2, s_2, s_3)^T = (15, 25, 35)^T$

Entra in base x_3.

Se $x_1 = \theta \geq 0$ e $x_1 = s_1 = 0$ (perché fuori base), abbiamo che:

$$\begin{aligned} x_2 + \tfrac{1}{2}\theta &= 15 \\ \tfrac{3}{2}\theta + s_2 &= 25 \\ \tfrac{1}{2}\theta + s_3 &= 35 \end{aligned} \qquad \Bigg\| \qquad \begin{aligned} x_2 &= 15 - \tfrac{1}{2}\theta \geq 0 \\ s_2 &= 25 - \tfrac{3}{2}\theta \geq 0 \\ s_3 &= 35 - \tfrac{1}{2}\theta \geq 0 \end{aligned}$$

$$\begin{cases} 15 - \frac{1}{2}\theta \geq 0 \\ 25 - \frac{3}{2}\theta \geq 0 \\ 35 - \frac{1}{2}\theta \geq 0 \end{cases} \qquad \begin{cases} \theta \leq 30 \\ \theta \leq \frac{50}{3} \\ \theta \leq 70 \end{cases}$$

Il massimo valore che θ può assumere è $\frac{50}{3}$. Tale valore annulla s_2 che esce dalla base.

La nuova base sarà $(x_2, x_3, s_3)^T$

$$\begin{array}{llllllllll}
 & z & + x_1 & & - x_3 & + s_1 & & & = 60 & (R0) \\
x_2 & & \frac{3}{4}x_1 & + x_2 & + \frac{1}{2}x_3 & + \frac{1}{4}s_1 & & & = 15 & (R1) \\
 & & \frac{5}{4}x_1 & & + \frac{3}{2}x_3 & - \frac{1}{4}s_1 & + s_2 & & = 25 & (R2) \\
s_3 & & -\frac{5}{4}x_1 & & + \frac{1}{2}x_3 & - \frac{3}{4}s_1 & & + s_3 & = 35 & (R3)
\end{array}$$

$R_2 := \frac{2}{3}R_2$

$$\begin{array}{llllllllll}
 & z & + x_1 & & - x_3 & + s_1 & & & = 60 & (R0) \\
x_2 & & \frac{3}{4}x_1 & + x_2 & + \frac{1}{2}x_3 & + \frac{1}{4}s_1 & & & = 15 & (R1) \\
 & & \frac{5}{6}x_1 & & + x_3 & - \frac{1}{6}s_1 & + \frac{2}{3}s_2 & & = \frac{50}{3} & (R2) \\
s_3 & & -\frac{5}{4}x_1 & & + \frac{1}{2}x_3 & - \frac{3}{4}s_1 & & + s_3 & = 35 & (R3)
\end{array}$$

$R_1 := R_1 - \frac{1}{2}R_2$; $R_3 := R_3 - \frac{1}{2}R_2$; $R_0 := R_0 + R_2$

$$\begin{array}{llllllll}
 & z + \frac{11}{6}x_1 & & & +\frac{5}{6}s_1 & +\frac{2}{3}s_2 & & = \frac{230}{3}\\
x_2 & \frac{1}{3}x_1 & +x_2 & & +\frac{1}{3}s_1 & & & = \frac{20}{3}\\
x_3 & \frac{5}{6}x_1 & & +x_3 & -\frac{1}{6}s_1 & +\frac{2}{3}s_2 & & = \frac{50}{3}\\
s_3 & -\frac{5}{3}x_1 & & & -\frac{2}{3}s_1 & -\frac{1}{3}s_2 & +s_3 & = \frac{80}{3}
\end{array}$$

Quindi $x_2^* = \frac{20}{3}$, $x_3^* = \frac{50}{3}$, $s_3^* = \frac{80}{3}$, $x_1^* = s_1^* = s_2^* = 0$

$$\underline{x}^* = (x_1^*, x_2^*, x_3^*, s_1^*, s_2^*, s_3^*)^T = \left(0, \frac{20}{3}, \frac{50}{3}, 0, 0, \frac{80}{3}\right)^T$$

$$z^* = \frac{230}{3}$$

TABLEAU

entra x_2

	z		x_1	x_2	x_3	s_1	s_2	s_3	F.O.	
	1	(R0)	-2	-4	-3	0	0	0	0	
s_1	0	(R1)	3	4	2	1	0	0	60	$\frac{60}{4}=15$
s_2	0	(R2)	2	1	2	0	1	0	40	$\frac{40}{1}=40$
s_3	0	(R3)	1	3	2	0	0	1	80	$\frac{80}{3}$

esce s_1 ; $R_1 := R_1/4$

	(R0)	1	-2	-4	-3	0	0	0	0
	(R1)	0	$\frac{3}{4}$	1	$\frac{1}{2}$	$\frac{1}{4}$	0	0	15
s_2	(R2)	0	2	1	2	0	1	0	40
s_3	(R3)	0	1	3	2	0	0	1	80

$R_0 := R_0 + 4R_1$

$R_2 := R_2 - R_1$

$R_3 := R_3 - 3R_1$

entra x_3

	(R0)	1	1	0	-1	1	0	0	60	
x_2	(R1)	0	$\frac{3}{4}$	1	$\frac{1}{2}$	$\frac{1}{4}$	0	0	15	$\frac{15}{\frac{1}{2}}=30$
s_2	(R2)	0	$\frac{5}{4}$	0	$\frac{3}{2}$	$-\frac{1}{4}$	1	0	25	$\frac{25}{\frac{3}{2}}=\frac{50}{3}$
s_3	(R3)	0	$-\frac{5}{4}$	0	$\frac{1}{2}$	$-\frac{3}{4}$	0	1	35	$\frac{35}{\frac{1}{2}}=70$

esce s_2 ; $R_2 := \frac{2}{3} R_2$

	(R0)	1	1	0	-1	1	0	0	60
x_2	(R1)	0	$\frac{3}{4}$	1	$\frac{1}{2}$	$\frac{1}{4}$	0	0	15
	(R2)	0	$\frac{5}{6}$	0	1	$-\frac{1}{6}$	$\frac{2}{3}$	0	$\frac{50}{3}$
s_3	(R3)	0	$-\frac{5}{4}$	0	$\frac{1}{2}$	$-\frac{3}{4}$	0	1	35

$R_0 := R_0 + R_2$

$R_1 := R_1 - \frac{1}{2} R_2$

$R_3 := R_3 - \frac{1}{2} R_2$

		z	x_1	x_2	x_3	s_1	s_2	s_3	F.O.
	(R0)	1	$\frac{11}{6}$	0	0	$\frac{5}{6}$	$\frac{2}{3}$	0	$\frac{230}{3}$
x_2	(R1)	0	$\frac{2}{3}$	1	0	$\frac{1}{3}$	$+\frac{1}{3}$	0	$\frac{20}{3}$
x_3	(R2)	0	$\frac{5}{6}$	0	1	$-\frac{1}{6}$	$\frac{2}{3}$	0	$\frac{50}{3}$
s_3	(R3)	0	$-\frac{5}{3}$	0	0	$-\frac{2}{3}$	$-\frac{1}{3}$	1	$\frac{80}{3}$

e si ritrova la soluzione ottima di prima.

LIEBERMAN, PAG. 105 ES. 12

in base

entra in base

$$\max \quad z = 4x_1 + 3x_2 + 6x_3$$
$$\text{s.t.} \quad 3x_1 + x_2 + 3x_3 \leq 30$$
$$2x_1 + 2x_2 + 3x_3 \leq 40$$
$$x_1, x_2, x_3 \geq 0$$

$\sim$

(0) $z - 4x_1 - 3x_2 - \underline{6x_3} = 0$

(1) y_1 $\quad 3x_1 + x_2 + 3x_3 + \underline{y_1} = 30$

(2) y_2 $\quad 2x_1 + 2x_2 + 3x_3 + \underline{y_2} = 40$

$x_1, x_2, x_3, y_1, y_2 \geq 0$

Sol. attuale: $(0, 0, 0, 30, 40)$, $z = 0$, base: $(y_1\ y_2)$

Se $x_3 = \theta \geq 0$

$y_1 = 30 - \cancel{3x_1} - \cancel{x_2} - 3\theta \geq 0$

$y_2 = 40 - \cancel{2x_1} - \cancel{2x_2} - 3\theta \geq 0$

$$\begin{cases} 30 - 3\theta \geq 0 \\ 40 - 3\theta \geq 0 \end{cases} \qquad \begin{cases} \theta \leq 10 \\ \theta \leq \frac{40}{3} \end{cases} \rightarrow \theta = 10$$

$\rightarrow y_1 = 0$ lascia la base

(0) $z - 4x_1 - 3x_2 - 6x_3 = 0$

(1) y_1 $\quad x_1 + \frac{1}{3}x_2 + \underline{x_3} + \frac{1}{3}y_1 = 10$

(2) y_2 $\quad 2x_1 + 2x_2 + 3x_3 + y_2 = 40$

$R0 + 6R1$; $R2 - 3R1$

$z + 2x_1 \underline{- x_2} + 2y_1 = 60$

$x_1 + \frac{1}{3}x_2 + \underline{x_3} + \frac{1}{3}y_1 = 10$

$-x_1 + x_2 - y_1 + \underline{y_2} = 10$

Sol. attuale: $(0, 0, 10, 0, 10)$, $z = 60$, base: (x_3, y_2) :

Se $x_2 = \theta \geq 0$:

$x_3 = 10 - \frac{1}{3}\theta \geq 0$

$y_2 = 10 - \theta \geq 0$

$$\begin{cases} 10 - \frac{1}{3}\theta \geq 0 \\ 10 - \theta \geq 0 \end{cases} \quad \begin{cases} \theta \leq 30 \\ \theta \leq 10 \end{cases} \longrightarrow \theta \leq 10$$

$\theta = 10$; $y_2 = 0$ lascia la base

$$\begin{array}{lllllllllll}
 & z & + 2x_1 & - x_2 & & + 2y_1 & & = & 60 & (0) \\
x_3 & & x_1 & + \frac{1}{3}x_2 & + x_3 & + \frac{1}{3}y_1 & & = & 10 & (1) \\
y_2 & & -x_1 & + \underline{x_2} & & - y_1 & + y_2 & = & 10 & (2)
\end{array}$$

$R_0 + R_2$; $R_1 - \frac{1}{3}R_2$

$$\begin{array}{lllllllll}
 & z & + x_1 & & & + y_1 & & = & 70 \\
x_3 & & \frac{4}{3}x_1 & & + x_3 & + \frac{2}{3}y_1 & - \frac{1}{3}y_2 & = & \frac{20}{3} \\
x_2 & & -x_1 & + x_2 & & - y_1 & + y_2 & = & 10
\end{array}$$

Sol. attuale: $(0, 10, \frac{20}{3}, 0, 0)$, $z = 70$, base: (x_3, x_2)

La soluzione trovata è ottima.

LIEBERMANN, PAG. 104. ES. 3.1.4

$$
\begin{array}{llr}
\max & z = 4500\,x_1 + 4500\,x_2 & \\
\text{s.t.} & x_1 \le 1 & (1) \\
 & x_2 \le 1 & (2) \\
 & 5000\,x_1 + 4000\,x_2 \le 6000 & (3) \\
 & 400\,x_1 + 500\,x_2 \le 600 & (4) \\
 & x_1,\ x_2 \ge 0 & (5)
\end{array}
$$

Per la risoluzione grafica riscrivo la f.o. così:

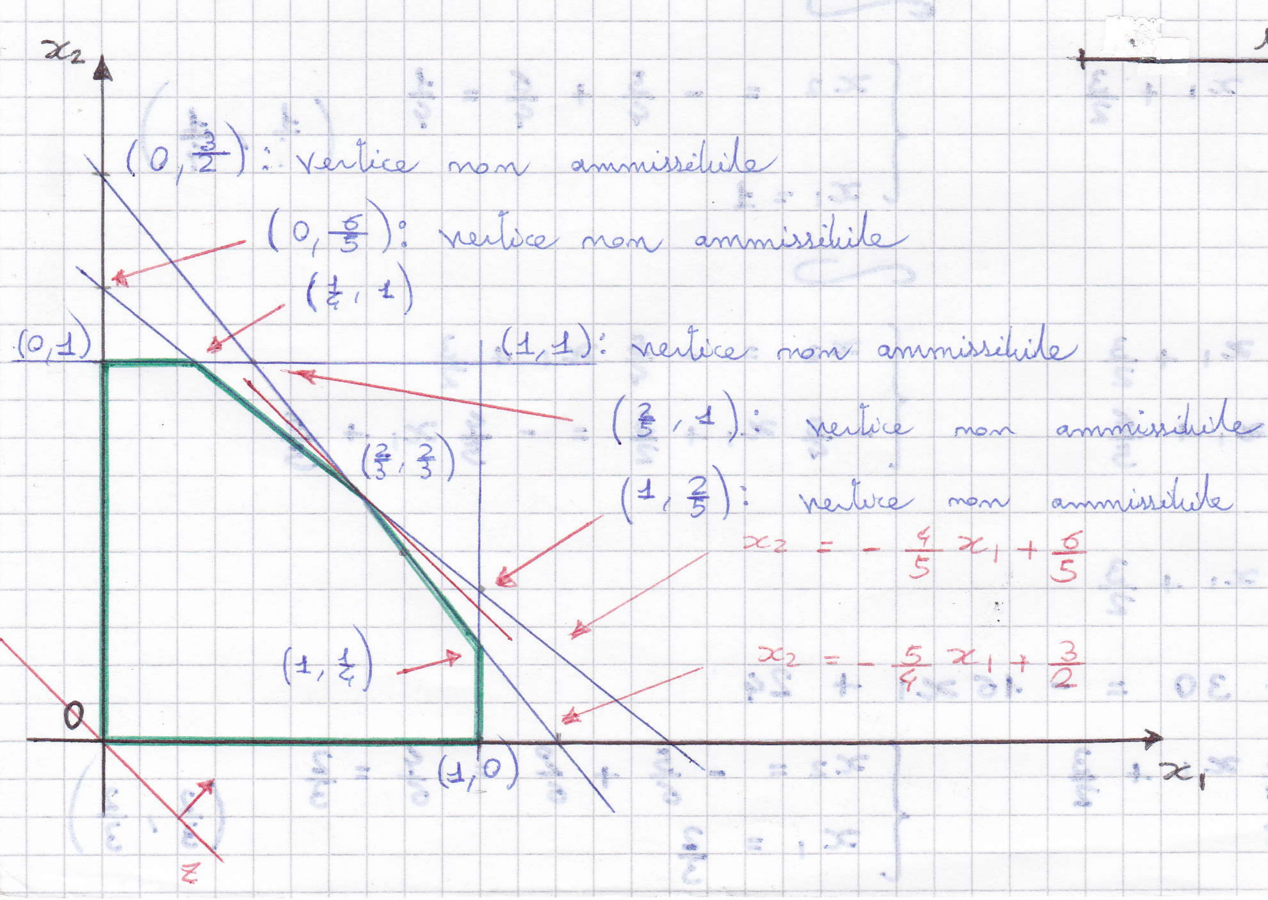
$$z = 4500\,(x_1 + x_2)$$

$$\frac{z}{4500} = x_1 + x_2$$

Pertanto posso massimizzare $z' = \frac{z}{4500} = x_1 + x_2$, e in ogni caso disegnare $x_2 = -x_1 + \frac{z}{4500}$. Riscrivo altresì i vincoli (3) e (4) nel seguente modo:

$5x_1 + 4x_2 \le 6$ (3') frontiera: $x_2 = -\frac{5}{4}x_1 + \frac{3}{2}$

$4x_1 + 5x_2 \le 6$ (4') frontiera: $x_2 = -\frac{4}{5}x_1 + \frac{6}{5}$



Determino i vertici non immediati.

$$\begin{cases} x_2 = -\frac{4}{5}x_1 + \frac{6}{5} \\ x_2 = 1 \end{cases} \qquad \begin{cases} 1 = -\frac{4}{5}x_1 + \frac{6}{5} \\ x_2 = 1 \end{cases}$$

$$\begin{cases} 5 = -4x_1 + 6 \\ x_2 = 1 \end{cases} \qquad \begin{cases} x_1 = \frac{1}{4} \\ x_2 = 1 \end{cases} \quad \left(\frac{1}{4}, 1\right)$$

$$\sim$$

$$\begin{cases} x_2 = -\frac{4}{5}x_1 + \frac{6}{5} \\ x_1 = 1 \end{cases} \qquad \begin{cases} x_2 = -\frac{4}{5} + \frac{6}{5} = \frac{2}{5} \\ x_1 = 1 \end{cases} \quad \left(1, \frac{2}{5}\right)$$

$$\sim$$

$$\begin{cases} x_2 = -\frac{5}{4}x_1 + \frac{3}{2} \\ x_2 = 1 \end{cases} \qquad \begin{cases} 1 = -\frac{5}{4}x_1 + \frac{3}{2} \\ x_2 = 1 \end{cases}$$

$$\begin{cases} \frac{5}{4}x_1 = \frac{1}{2} \\ x_2 = 1 \end{cases} \qquad \begin{cases} x_1 = \frac{2}{5} \\ x_2 = 1 \end{cases} \quad \left(\frac{2}{5}, 1\right)$$

$$\sim$$

$$\begin{cases} x_2 = -\frac{5}{4}x_1 + \frac{3}{2} \\ x_1 = 1 \end{cases} \qquad \begin{cases} x_2 = -\frac{5}{4} + \frac{6}{4} = \frac{1}{4} \\ x_1 = 1 \end{cases} \quad \left(1, \frac{1}{4}\right)$$

$$\sim$$

$$\begin{cases} x_2 = -\frac{5}{4}x_1 + \frac{3}{2} \\ x_2 = -\frac{4}{5}x_1 + \frac{6}{5} \end{cases} \qquad \begin{cases} x_2 = -\frac{5}{4}x_1 + \frac{3}{2} \\ -\frac{5}{4}x_1 + \frac{3}{2} = -\frac{4}{5}x_1 + \frac{6}{5} \end{cases}$$

$$\begin{cases} x_2 = -\frac{5}{4}x_1 + \frac{3}{2} \\ -25x_1 + 30 = -16x_1 + 24 \end{cases}$$

$$\begin{cases} x_2 = -\frac{5}{4}x_1 + \frac{3}{2} \\ 9x_1 = 6 \end{cases} \qquad \begin{cases} x_2 = -\frac{5}{6} + \frac{9}{6} = \frac{4}{6} = \frac{2}{3} \\ x_1 = \frac{2}{3} \end{cases} \quad \left(\frac{2}{3}, \frac{2}{3}\right)$$

Verifica

$$-\frac{4}{5}\cdot\frac{2}{3}+\frac{6}{5} = \frac{2}{5}\left(-\frac{4}{3}+3\right) = \frac{2}{5}\cdot\frac{5}{3} = \frac{2}{3}$$

$$-\frac{5}{4}\cdot\frac{2}{3}+\frac{3}{2} = -\frac{5}{6}+\frac{9}{6} = \frac{4}{6} = \frac{2}{3}$$ Ok!

VERTICE	F.O.	AMMISSIBILE
$(0,0)$	0	sì
$(0,1)$	4500	sì
$\left(0,\frac{6}{5}\right)$	$4500\cdot\frac{6}{5} = 450\cdot 12 =$ $= 450(10+2) = 4500+900 =$ $= 5500-100 = 5400$	no
$\left(0,\frac{3}{2}\right)$	$4500\cdot\frac{3}{2} = 45\cdot 3\cdot 50 =$ $= 45\cdot 150 = 4500+45\cdot 50 =$ $= 4500+450\cdot 5 =$ $= 4500+\frac{4500}{2} =$ $= 4500+\frac{4000+500}{2} =$ $= 4500+2000+250 =$ $= 6500+250 = 6750$	no
$\left(\frac{1}{4},1\right)$	$4500\left(\frac{1}{4}+1\right) = 4500+\frac{4500}{4} =$ $= 4500+\frac{4000+500}{4} =$ $= 4500+1000+\frac{250}{2} =$ $= 5500+\frac{200+50}{2} =$ $= 5500+100+25 = 5625$	sì
$\left(\frac{2}{5},1\right)$	$4500\cdot\frac{2}{5}+4500 =$ $= 450\cdot 4+4500 =$ $= 1800+4500 = 6300$	no

VERTICE	F.O.	AMMISSIBILE
$\left(\frac{2}{3}, \frac{2}{3}\right)$	$4500 \cdot \frac{4}{3} = 1500 \cdot 4 = 6000$	sí
$(1, 1)$	8000	no
$\left(1, \frac{2}{5}\right)$	$4500 + 4500 \cdot \frac{2}{5} =$ $= 4500 + 1800 =$ $= 5500 + 800 = 6300$	no
$\left(1, \frac{1}{4}\right)$	come $\left(\frac{1}{4}, 1\right)$: 5625	sí
$(1, 0)$	come $(0, 1)$: 4500	sí

Pertanto la soluzione ottima è il vertice $\left(\frac{2}{3}, \frac{2}{3}\right)$, cui corrisponde una funzione obiettivo pari a 6000

Le soluzioni possibili del metodo del simplesso sono le seguenti:

$$(0,0) \rightarrow (0,1) \rightarrow \left(\frac{1}{4}, 1\right) \rightarrow \left(\frac{2}{3}, \frac{2}{3}\right), \text{ oppure}$$

$$(0,0) \rightarrow (1,0) \rightarrow \left(1, \frac{1}{4}\right) \rightarrow \left(\frac{2}{3}, \frac{2}{3}\right)$$

FORMA STANDARD

$$\begin{array}{llllllllll}
\max & \frac{z}{4500} & - x_1 & - x_2 & & & & & = & 0 \\
\text{s. t.} & & x_1 & & + y_1 & & & & = & 1 \\
 & & & x_2 & & + y_2 & & & = & 1 \\
 & & 5x_1 & + 4x_2 & & & + y_3 & & = & 6 \\
 & & 4x_1 & + 5x_2 & & & & + y_4 & = & 6 \\
 & & \multicolumn{6}{l}{x_1, x_2, y_1, y_2, y_3, y_4 \geq 0} & &
\end{array}$$

TABLEAU

		z'	x_1	x_2	y_1	y_2	y_3	y_4			
	R_0	1	-1	-1	0	0	0	0	0		$R_0 + R_1$

IT. 0

y_1	R_1	0	1	0	1	0	0	0	1	$\frac{1}{1} = 1$	
y_2	R_2	0	0	1	0	1	0	0	1		
y_3	R_3	0	5	4	0	0	1	0	6	$\frac{6}{5}$	$R_3 - 5R_1$
y_4	R_4	0	4	5	0	0	0	1	6	$\frac{6}{4}$	$R_4 - 4R_1$

B^{-1}

IT. 1

	R_0	1	0	-1	1	0	0	0	1		$R_0 + R_3'$
x_1	R_1	0	1	0	1	0	0	0	1		
y_2	R_2	0	0	1	0	1	0	0	1	$\frac{1}{1}$	$R_2 - R_3'$
y_3	R_3	0	0	4	-5	0	1	0	1	$\frac{1}{4}$	$R_3' = \frac{R_3}{4}$
y_4	R_4	0	0	5	-4	0	0	1	2	$\frac{2}{5}$	$R_4 - 5R_3'$

B^{-1}

IT. 2

	R_0	1	0	0	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	$\frac{5}{4}$		$R_0 + \frac{1}{4} R_4'$
x_1	R_1	0	1	0	1	0	0	0	1	1	$R_1 - R_4'$
y_2	R_2	0	0	0	$\frac{5}{4}$	1	$-\frac{1}{4}$	0	$\frac{3}{4}$	$\frac{3}{5}$	$R_2 - \frac{5}{4} R_4'$
x_2	R_3	0	0	1	$-\frac{5}{4}$	0	$\frac{1}{4}$	0	$\frac{1}{4}$		$R_3 + \frac{5}{4} R_4'$
y_4	R_4	0	0	0	$\frac{9}{4}$	0	$-\frac{5}{4}$	1	$\frac{3}{4}$	$\frac{1}{3}$	$R_4' = \frac{4}{9} R_4$

B^{-1}

IT. 3

	R_0	1	0	0	0	0	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{4}{3}$
x_1	R_1	0	1	0	0	0	$\frac{5}{9}$	$-\frac{4}{9}$	$\frac{2}{3}$
y_2	R_2	0	0	0	0	1	$\frac{4}{9}$	$-\frac{5}{9}$	$\frac{1}{3}$
x_2	R_3	0	0	1	0	0	$-\frac{4}{9}$	$\frac{5}{9}$	$\frac{2}{3}$
y_1	R_4	0	0	0	1	0	$-\frac{5}{9}$	$\frac{4}{9}$	$\frac{1}{3}$

B^{-1}

La soluzione ottima è $(x_1, x_2) = \left(\frac{2}{3}, \frac{2}{3}\right)$, e la funzione obiettivo originale ottima è:

$z = 4500\, z' = 4500 \cdot \frac{4}{3} = 6000$, come già precedentemente trovato.

SIMPLESSO IN FORMA MATRICIALE

IT. 0

$$\underline{x}_B = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \qquad \underline{\underline{B}} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} = \underline{\underline{B}}^{-1} \qquad \underline{b} = \begin{bmatrix} 1 \\ 1 \\ 6 \\ 6 \end{bmatrix}$$

$$\underline{x}_B = \underline{\underline{B}}^{-1} \underline{b} = \begin{bmatrix} 1 \\ 1 \\ 6 \\ 6 \end{bmatrix}$$

$$\underline{c}_B^T = [0, 0, 0, 0]$$

$$z' = \underline{c}_B^T \, \underline{x}_B = 0$$

IT. 1

$$\underline{x}_B = \begin{bmatrix} x_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \qquad \underline{\underline{B}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 5 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 \end{bmatrix} \qquad \underline{\underline{B}}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -5 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \end{bmatrix}$$

Verifica:

$$\underline{\underline{B}} \, \underline{\underline{B}}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 5 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -5 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{B}}^{-1} \underline{\underline{B}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -5 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 5 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{x}_B = \underline{\underline{B}}^{-1} \underline{b} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -5 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

$$c_B^T = [1, 0, 0, 0]$$

$$z' = \underline{c}_B^T \, \underline{x}_B = [1, 0, 0, 0] \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} = 1$$

IT. 2

$$\underline{x}_B = \begin{bmatrix} x_1 \\ y_2 \\ x_2 \\ y_4 \end{bmatrix} \quad \underline{\underline{B}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 5 & 0 & 4 & 0 \\ 4 & 0 & 5 & 1 \end{bmatrix} \quad \underline{\underline{B}}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{5}{4} & 1 & -\frac{1}{4} & 0 \\ -\frac{5}{4} & 0 & \frac{1}{4} & 0 \\ \frac{9}{4} & 0 & -\frac{5}{4} & 1 \end{bmatrix}$$

Verifica:

$$\underline{\underline{B}}\,\underline{\underline{B}}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 5 & 0 & 4 & 0 \\ 4 & 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{5}{4} & 1 & -\frac{1}{4} & 0 \\ -\frac{5}{4} & 0 & \frac{1}{4} & 0 \\ \frac{9}{4} & 0 & -\frac{5}{4} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{B}}^{-1}\underline{\underline{B}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{5}{4} & 1 & -\frac{1}{4} & 0 \\ -\frac{5}{4} & 0 & \frac{1}{4} & 0 \\ \frac{9}{4} & 0 & -\frac{5}{4} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 5 & 0 & 4 & 0 \\ 4 & 0 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{x}_B = \underline{\underline{B}}^{-1}\underline{b} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{5}{4} & 1 & -\frac{1}{4} & 0 \\ -\frac{5}{4} & 0 & \frac{1}{4} & 0 \\ \frac{9}{4} & 0 & -\frac{5}{4} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{3}{4} \\ \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$$

$$\underline{c}_B^T = [1, 0, 1, 0]$$

$$z' = \underline{c}_B^T \underline{x}_B = [1, 0, 1, 0] \begin{bmatrix} 1 \\ \frac{3}{4} \\ \frac{1}{4} \\ \frac{3}{4} \end{bmatrix} = 1 + \frac{1}{4} = \frac{5}{4}$$

IT. 3

$$\underline{x}_B = \begin{bmatrix} x_1 \\ y_2 \\ x_2 \\ y_1 \end{bmatrix} \quad \underline{\underline{B}} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 5 & 0 & 4 & 0 \\ 4 & 0 & 5 & 0 \end{bmatrix} \quad \underline{\underline{B}}^{-1} = \begin{bmatrix} 0 & 0 & \frac{5}{9} & -\frac{4}{9} \\ 0 & 1 & \frac{4}{9} & -\frac{5}{9} \\ 0 & 0 & -\frac{4}{9} & \frac{5}{9} \\ 1 & 0 & -\frac{5}{9} & \frac{4}{9} \end{bmatrix}$$

$$\underline{\underline{B}}\,\underline{\underline{B}}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 5 & 0 & 4 & 0 \\ 4 & 0 & 5 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & \frac{5}{9} & -\frac{4}{9} \\ 0 & 1 & \frac{4}{9} & -\frac{5}{9} \\ 0 & 0 & -\frac{4}{9} & \frac{5}{9} \\ 1 & 0 & -\frac{5}{9} & \frac{4}{9} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{B}}^{-1}\underline{\underline{B}} = \begin{bmatrix} 0 & 0 & \frac{5}{9} & -\frac{4}{9} \\ 0 & 1 & \frac{4}{9} & -\frac{5}{9} \\ 0 & 0 & -\frac{4}{9} & \frac{5}{9} \\ 1 & 0 & -\frac{5}{9} & \frac{4}{9} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 5 & 0 & 4 & 0 \\ 4 & 0 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{x}_B = \underline{\underline{B}}^{-1}\underline{b} = \begin{bmatrix} 0 & 0 & \frac{5}{9} & -\frac{4}{9} \\ 0 & 1 & \frac{4}{9} & -\frac{5}{9} \\ 0 & 0 & -\frac{4}{9} & \frac{5}{9} \\ 1 & 0 & -\frac{5}{9} & \frac{4}{9} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 6 \\ 6 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$\underline{c}_B^T = [1, 0, 1, 0]$$

$$z' = \underline{c}_B^T \underline{x}_B = [1, 0, 1, 0] \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} = \frac{4}{3}$$