Application Optimal risky portfolios

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Application 1

- Find the optimal risky portfolio formed from 8 country index portfolios, using the data provided below.
- What are the mean and the variance of the portfolio's rate of return?

index portfolios	I1	I2	I3	I4	I5	I6	I7	I8
Mean Return	15.5393	6.3852	26.5999	1.4133	18.0745	16.6347	16.2243	17.2306
Standard Deviation	26.4666	41.1475	26.0514	26.0709	21.6918	25.0779	38.7686	17.1944

	Correlation Matrix							
	I1	I2	I3	I4	I5	I6	I7	I8
I1	1.0000	0.4496	0.6062	0.4539	0.5898	0.6977	0.5427	0.5665
I2	0.4496	1.0000	0.3399	0.6295	0.5039	0.5753	0.7752	0.6672
I3	0.6062	0.3399	1.0000	0.1503	0.6867	0.7125	0.4711	0.4002
I4	0.4500	0.6295	0.1503	1.0000	0.4951	0.4287	0.4852	0.6066
I5	0.5898	0.5039	0.6867	0.4951	1.0000	0.7708	0.4633	0.6294
I6	0.6977	0.5753	0.7125	0.4287	0.7708	1.0000	0.5406	0.6378
I7	0.5427	0.7752	0.4711	0.4852	0.4633	0.5406	1.0000	0.7243
I8	0.5665	0.6672	0.4002	0.6066	0.6294	0.6378	0.7243	1.0000

Application 1

Theoretical aspects

- The optimal risky portfolio represents the optimal combination of risky assets that maximizes the Sharpe ratio.
- Finding the optimal risky portfolio means finding the tangency point of Capital Allocation Line to the Efficient Frontier of risky assets
- The first step in finding the optimal risky portfolio is to calculate the minimum-variance portfolio and to plot the minimum –variance frontier of risky portfolio.
- The minimum-variance frontier is defined by all pairs [E(r); $\boldsymbol{\sigma}$] of possible minimum variance portfolios.
- Using the expected returns, variances and covariance matrix of risky assets we can calculate the minimum-variance portfolio consistent with the minimum variance for any targeted expected return.

Application 1

Solution

- Bordered covariance matrix will be used to calculate the portfolio variance. Therefore, the first step is to determine the covariance matrix of the 8 index portfolios, knowing that

$$Cov(r_i, r_j) = \rho_{i,j}\sigma_i\sigma_j$$

Substituting the values for correlation coefficient, and standard deviations of risky assets in the above formula, we obtain the covariance matrix:

	I1	I2	I3	I4	I5	I6	I7	I8
I1	700.4814	489.5784	417.9441	313.1641	338.6342	463.1149	556.8682	257.8227
I2	489.5784	1693.1161	364.3733	675.2964	449.7674	593.6221	1236.5777	472.0434
I3	417.9441	364.3733	678.6757	102.0882	388.0346	465.4811	475.7823	179.2474
I4	313.1641	675.2964	102.0882	679.6933	280.0137	280.3160	490.4024	271.9451
I5	338.6342	449.7674	388.0346	280.0137	470.5337	419.3265	389.6314	234.7461
I6	463.1149	593.6221	465.4811	280.3160	419.3265	628.9036	525.5507	275.0026
I7	556.8682	1236.5777	475.7823	490.4024	389.6314	525.5507	1503.0060	482.8325
I8	257.8227	472.0434	179.2474	271.9451	234.7461	275.0026	482.8325	295.6475

Application 1

Solution

2. We need to calculate the minimum-variance portfolio and to plot the minimum –variance frontier of risky portfolio.

- The minimum-variance frontier is the set of portfolios that minimize the risk for any targeted portfolio expected return.
- we consider for the beginning an equally weighted portfolio wI1=wI2=wI3=wI4=wI5=wI6=wI7=wI8=$\frac{1}{8} = 0.125$
- All calculations are made using Excel functions
- The portfolio variance was determined by using the bordered covariance matrix, considering the weights of each index portfolio, that verifies the generalized formula for n assets of $\sigma_p^2 = w_i^2\sigma_i^2 + w_j^2\sigma_j^2 + 2w_i \ w_j \ \sigma_i \ \sigma_j \ \rho_{i,j}$
- By using matrix algebra, portfolio variance is determined using the covariance matrix, the matrix of weights, W, and its transpose, W^T: $\sigma_p^2 = W^T\Sigma W$

Application 1

Solution

	Bordered Covariance Matrix: Equally Weighted Portfolio							
Weights	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125
0.125	10.9450	7.6497	6.5304	4.8932	5.2912	7.2362	8.7011	4.0285
0.125	7.6497	26.4549	5.6933	10.5515	7.0276	9.2753	19.3215	7.3757
0.125	6.5304	5.6933	10.6043	1.5951	6.0630	7.2731	7.4341	2.8007
0.125	4.8932	10.5515	1.5951	10.6202	4.3752	4.3799	7.6625	4.2491
0.125	5.2912	7.0276	6.0630	4.3752	7.3521	6.5520	6.0880	3.6679
0.125	7.2362	9.2753	7.2731	4.3799	6.5520	9.8266	8.2117	4.2969
0.125	8.7011	19.3215	7.4341	7.6625	6.0880	8.2117	23.4845	7.5443
0.125	4.0285	7.3757	2.8007	4.2491	3.6679	4.2969	7.5443	4.6195
1.000	55.2751	93.3496	47.9942	48.3269	46.4170	57.0518	88.4477	38.5826
Portfolio Variance		475.4449						
Portfolio Standard Deviation		21.8046984						
Portfolio Mean		14.7627179						

- The bordered covariance matrix in the case of equally weighted portfolio, with the corresponding portfolio's variance, standard deviation and expected return.

Application 1

Solution

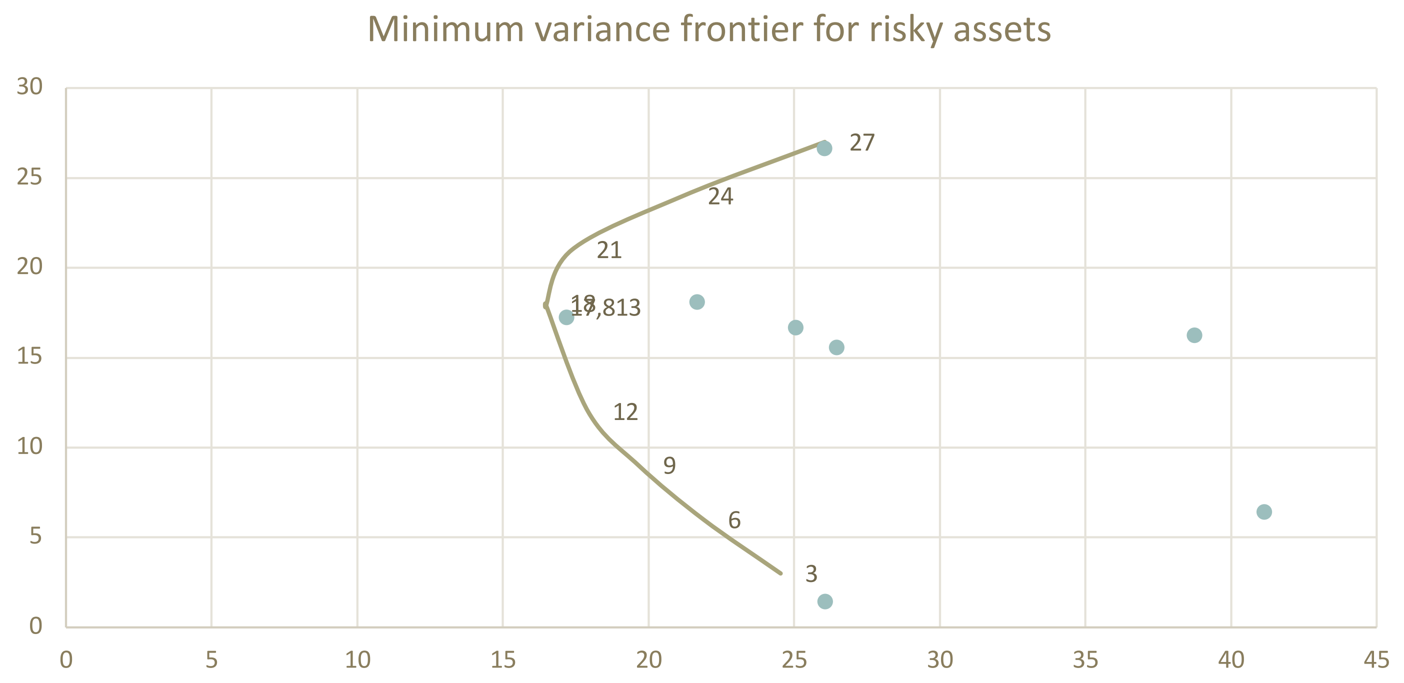
- To obtain different E(r)-SD pairs for the minimum-variance portfolios of the 8 risky assets in order to determine the minimum-variance frontier, it will be used Excel solver function.
- The objective function is min SD, with the constraints that the total weights of risky =1, and different targeted expected returns.
- In establishing the targeted expected returns we start from the lower expected return that is offered by the risky assets (1.385) until the highest one (26.599),so from 1until 27 with intermediate values increased by 300 basis point.
- We will get all the minimum variance portfolios for the targeted expected returns that plot the minimum variance frontier of risky assets.
- The portfolio weights obtained, as well the corresponding SD for the targeted returns are included in the next table

Application 1

Solution

all the pairs Mean-St.Dev plot the minimum variance frontier.

Mean	St. Dev	wl1	wl2	wl3	wl4	wl5	wl6	wl7	wl8
3.00	24.5365	0.0016	0	0	0.8986	0	0.0221	0	0.0775
6.00	21.8883	0.0165	0.0000	0.0000	0.7075	0.0000	0.0188	0.0000	0.2571
9.00	19.6644	0.0217	0.0000	0.0185	0.5300	0.0176	0.0000	0.0000	0.4122
12.00	17.9265	0.0000	0.0000	0.0826	0.3805	0.0171	0.0000	0.0000	0.5198
15.00	16.8147	0.0000	0.0000	0.1372	0.2232	0.0168	0.0000	0.0000	0.6229
18.00	16.4614	0.0000	0.0000	0.1918	0.0658	0.0164	0.0000	0.0000	0.7260
21.00	17.3685	0.0000	0.0000	0.4023	0.0000	0.0000	0.0000	0.0000	0.5977
24.00	21.1878	0.0000	0.0000	0.7225	0.0000	0.0000	0.0000	0.0000	0.2775
27.00	26.0514	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000



Minimum variance frontier for risky assets
30
25
20
15
10
5
0
0
5
10
15
20
25
30
35
40
45
27
24
21
18
17,813
12
9
6
3

Application 1

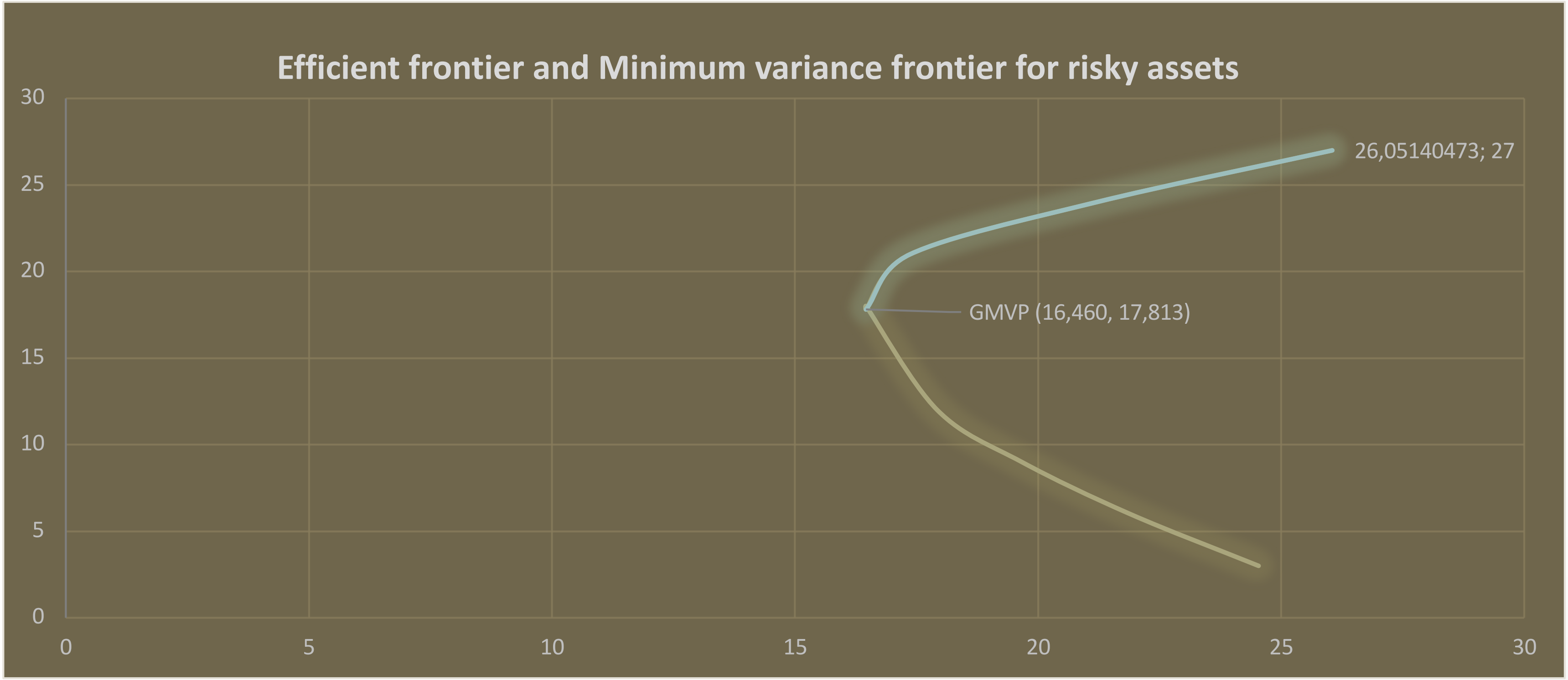
Solution

- The next step is to determine the global minimum-variance portfolio (GMVP), which represent the starting point of the Efficient frontier of risky assets.
- Using Excel solve function with the objective function min(SD), and the constraint total assets weights =1, we will obtain the weight of each risky asset that form the GMVP, and portfolio's expected return and variance.

GMVP weights	0
	0
	0.188374
	0.07564466
	0.016453
	0
	0
	0.71952834
	1
variance	270.925587
sd	16.4598173
Er	17.8128798

Application 1

Solution



Application 1

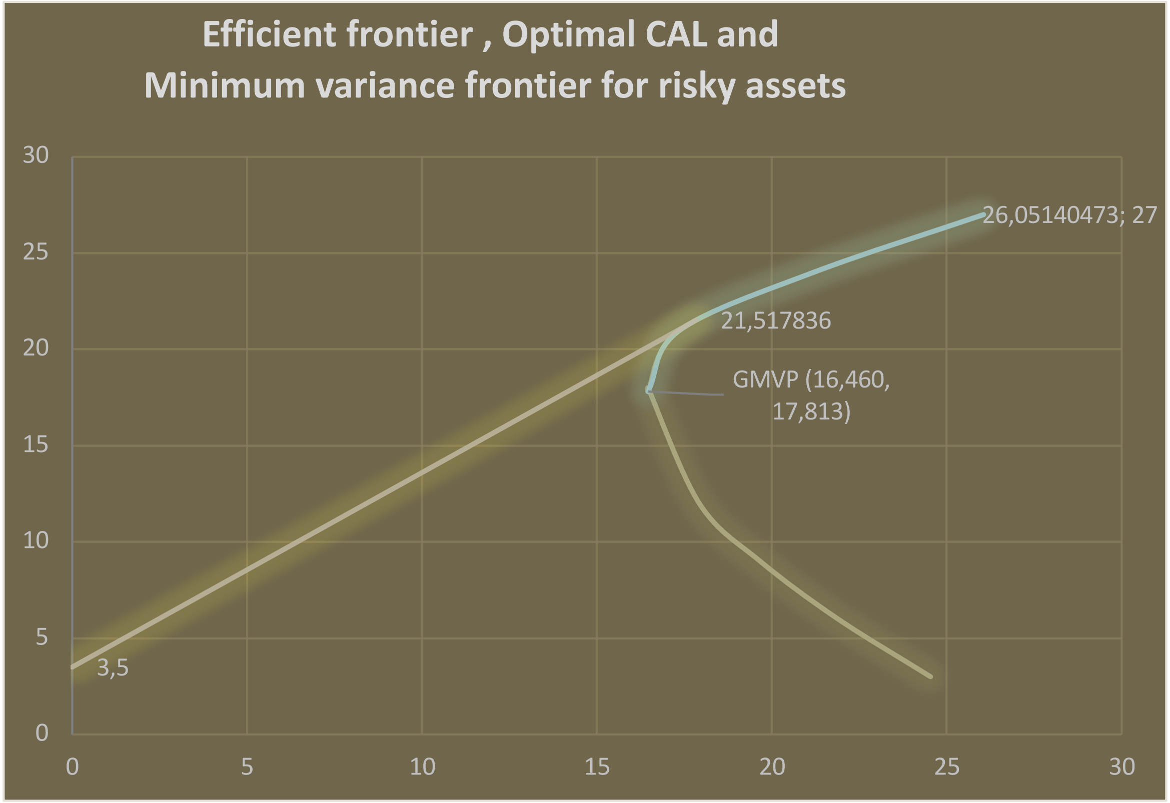
Solution

- Having the efficient frontier, we have to determine the tangency point of CAL to the frontier in order to identify the optimal risky portfolio.
- By using Solve function, with object function the maximization of Sharpe ratio and with the constraint of the total weights of risky assets being equal to 1, we will obtain the weights of risky assets that can form a pair of expected returns and standard deviation that represents the optimal risky portfolio
- Before using Solve function, we need to determine the Expected Excess Returns, considering that risk-free return is 3.5%
- Then, considering the Sharpe ratio calculated for the GMVP, we will apply the Solve function, as mentioned above, and will obtain the weights of the optimal risk portfolio (ORP), as well as its variance, standard deviation and expected return.

Application 1

Solution

We can plot the CAL and we have the tangency point on Efficient frontier



ORPweights	0
	0
	0.45758784
	0
	0
	0
	0
	0.54241216
	1
variance	318.067165
sd	17.8344376
Er	21.517836
sharpe ratio	1.01028338

Application 1

Solution

- The final solution:
- The optimal risky portfolio has 2 risky assets, respectively 45.75% is represented by I3 and 54.24% by I8 .

The next step should be the one regarding the investor's choice for the appropriate mix between the risk portfolio and the risk-free asset.