International diversification

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Lecture Overview

- International Diversification in Portfolio Theory
- Risk, Return, and Global Portfolio Construction

Mean-Variance Framework

- Goal:
  - maximize return for given risk

    OR
- minimize risk for a given return

- Portfolio return = weighted sum of assets

$$E(r_p) = \sum_{i=1}^{N} w_i E(r_i)$$

- Risk measured by variance $\sigma_p^2 = \Sigma\Sigma w_i w_j \sigma_{ij}$
  - Where σ_{ij} is the covariance and $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$

Portfolio Variance Formula

- Example for two risky assets *a* and *b*
- $\sigma_P^2 = w_a^2\sigma_a^2 + w_b^2\sigma_b^2 + 2w_a w_b cov(a,b)$
- *Cov(a,b) :* $\sigma_{ab} = \rho_{a,b}\sigma_a\sigma_b$, *where* $\rho_{a,b}$ *- correlation coefficient*
- If $\rho_{a,b}$ = +1 (perfect positive correlation), there is no diversification benefit.
- If $\rho_{a,b}$ = 0, portfolio variance equals the sum of weighted squared variances.
- If $\rho_{a,b}$ = −1, it is theoretically possible to construct a zero-variance portfolio

Diversification Principle

- Correlation < 1 reduces risk
- International assets expand frontier

Numerical example: Two country -portfolio

Consider an investor domiciled in the eurozone, with access to two equity indices: Italy (IT) with an expected annual return of 8% and standard deviation of 20%, and the United States (US) with an expected annual return of 10% and standard deviation of 18%. The correlation between the two markets is $\rho = 0.55$.

a) Analyze (in terms of mean and variance) portfolios formed by combining the two markets in varying proportions
b) Identify the minimum variance3 portfolio

Numerical example: Two country –portfolio (II)

Weight Italy (w_IT)	Weight USA (w_US)	$E(R_p)$ %	σ_p %
100%	0%	8.00	20.00
80%	20%	8.40	18.46
60%	40%	8.80	17.45
40%	60%	9.20	16.60
20%	80%	9.60	17.76
0%	100%	10.00	18.00

a) Example of calculation for w_IT = 40%, w_US = 60%:

$\sigma^2_p = (0.40)^2(0.20)^2 + (0.60)^2(0.18)^2 + 2(0.40)(0.60)(0.55)(0.20)(0.18) = 0.0064 + 0.011664 + 0.009504 = 0.027568$

$\sigma_p = \sqrt{0.027568} \approx 16.60\%$

b) Minimum variance portfolio

$$w^*_{IT} = \frac{\sigma^2_{US} - \rho_{IT,US}\sigma_{IT}\sigma_{US}}{\sigma^2_{IT} + \sigma^2_{US} - 2\rho_{IT,US}\sigma_{IT}\sigma_{US}}$$

w*_IT = (0.0324 − 0.55 × 0.20 × 0.18) / (0.04 + 0.0324 − 2×0.55×0.20×0.18)

w*_IT = 0.0126 / 0.0328 ≈ 38.4%, w*_US = 61.6%

International extension of Portfolio Theory

International CAPM

- The standard CAPM (Sharpe, 1964; Lintner, 1965) prices assets in a single-currency integrated market

$$E(r_i) = r_f + \beta_i \cdot [E(r_m) - r_f]$$

- When extended to the international context, (Solnik (1974) and Adler and Dumas (1983)) the model becomes ICAPM, which introduces currency risk as an additional priced factor.
- In a world of N currencies, expected excess returns are determined both by exposure to the world market portfolio and by exposure to exchange rate risk:

$$E(r_i) - r_f = \beta_{i,w} \cdot [E(r_w) - r_f) + \Sigma_k \gamma_{ik} \cdot \lambda_k, \quad \text{where}$$

$\beta_{i,w}$ - the beta with respect to the world market portfolio,

γ_{ik} - the currency risk exposure of asset i to currency k,

λ_k - the currency risk premium.

International CAPM (II)

The ICAPM predicts that:

- in integrated international capital markets, only world systematic risk is priced;
- country-specific idiosyncratic risk can be diversified away.

However, the result depends on assumptions

- perfectly integrated markets,
- no barriers,
- purchasing power parity

that are consistently violated in practice

Barriers to Diversification

- Taxes, regulation, info asymmetry
- Transaction costs
- Home bias

Currency Risk (Exchange rate risk)

The return to a domestic investor from holding a foreign asset can be decomposed as follows.

Let r_l be the local-currency return and *e* the percentage change in the exchange rate.

The return in domestic currency is approximately:

$$r^d \approx r_l + e$$

The variance of the domestic-currency return is:

$$Var(r^d) = Var(r_l) + Var(e) + 2 \cdot Cov(r_l, e)$$

Concl:

currency risk adds to total risk, while the covariance term can either amplify or reduce risk depending on whether the currency and the local equity market tend to move together or in opposite directions.

Currency Risk (Exchange rate risk) – numerical example

EUR/USD Currency Effect

An European investor holds US equities.

In Year 1:

US equity return (USD) = +12%;

USD depreciates 5% against EUR

→ domestic (EUR) return ≈ +7%.

In Year 2:

US equity return (USD) = −8%;

USD appreciates 3%

→ domestic (EUR) return ≈ −5%.

The currency effect is not passive—it introduces an additional source of variance that must be explicitly managed.

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Hedging Currency Risk

Currency risk can be mitigated through forward contracts, currency futures, options, or currency overlay strategies.

The cost of a forward hedge is determined by covered interest rate parity:

$$\frac{F}{S} = \frac{1+r_d}{1+r_f}, \qquad \text{where}$$

- F - the forward rate,
- S - the spot rate,
- r_d - the domestic risk-free rate
- r_f - the foreign risk-free rate.

Country Risk

Investing internationally exposes portfolios to country-specific risks:

- **Political risk** - encompasses policy changes, expropriation, nationalisation, political instability, or changes in the regulatory environment
- **Sovereign risk** - refers to the possibility of government default on debt obligations.
- **Liquidity risk** - is elevated in emerging and frontier markets with shallower trading volumes
- **Contagion risk** - arises because financial crises spread across borders, dramatically raising cross-market correlations when diversification is most needed

Country Risk Premium Estimation

- Country risk premium (CRP) approach adjusts the CAPM for country-specific risk:

$$E(r_i) = r_f + \beta \cdot [E(r_m) - r_i] + CRP,$$

- where CRP is estimated as:

$$CRP = Sovereign\ Sprea \times \frac{\sigma_{Equity}}{\sigma_{Bond}}$$

Example:

If a country has a sovereign spread of 200 basis points and the equity-to-bond volatility ratio is 1.5, the CRP equals 3.0%.

Correlation Structure

- Developed: 0.5–0.85
- Emerging: 0.3–0.65

Time-Varying Correlations

- Correlations rise in crises
- Reduce diversification benefits

Multi-Asset International Portfolio

Four country portfolio construction

Consider an investor constructing a globally diversified equity portfolio using four markets with the following characteristics (expected returns, standard deviations and correlation matrix):

Market	Expected Return (%)	Standard Deviation (%)
USA	10.0	18.0
Eurozone (EZ)	8.5	20.0
Japan (JP)	7.5	22.0
Emerging Markets (EM)	12.0	28.0

	USA	EZ	Japan	EM
USA	1.00	0.68	0.42	0.55
EZ	0.68	1.00	0.45	0.52
Japan	0.42	0.45	1.00	0.40
EM	0.55	0.52	0.40	1.00

Global Portfolio Example

- Equal-weight portfolio
- Improved Sharpe ratio

Equal-Weight Portfolio

For the equally weighted portfolio (w_i = 25% each):

$$E(R_p) = 0.25(10.0 + 8.5 + 7.5 + 12.0) = 9.50\%$$

Computing the full variance-covariance contribution:

Diagonal variance terms: USA = 0.002025, EZ = 0.002500, JP = 0.003025, EM = 0.004900 → Sum = 0.01245

Off-diagonal covariance terms (all pairs multiplied by $2w_iw_j$ = 0.125):

Cov (USA,EZ) = 0.125 × 0.02448 = 0.003060

Cov (USA,JP) = 0.125 × 0.016632 = 0.002079

Cov (USA,EM) = 0.125 × 0.027720 = 0.003465

Cov (EZ,JP) = 0.125 × 0.019800 = 0.002475

Cov (EZ,EM) = 0.125 × 0.029120 = 0.003640

Cov(JP,EM) = 0.125 × 0.024640 = 0.003080

Sum off-diag = 0.017799

$$\sigma^2_p = 0.01245 + 0.017799 = 0.030249 \quad \rightarrow \quad \sigma_p \approx 17.39\%$$

The equal-weight global portfolio achieves an expected return of 9.50% with a standard deviation of approximately 17.4%—significantly superior to the Eurozone-only portfolio (8.5%; 20.0%) on both dimensions simultaneously.

Sharpe Ratio Comparison (Rf = 2.5%)

Portfolio	$E(R_p)$	σ_p	Sharpe Ratio
Eurozone only	8.50%	20.00%	0.30
USA only	10.00%	18.00%	0.42
Equal-weight global	9.50%	17.39%	0.40
Optimised (min-var, approx.)	~9.0%	~15.8%	~0.41

Sharpe = (Rp − Rf)/σp

Conclusion

- Diversification works but imperfect
- Requires active management