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A NOTE ON COMPETITIVE BRIBERY GAMES *

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Within a competitive bribery game in which each player has incomplete information, we show that there exists a unique Nash equilibrium which is symmetric. Similar results hold for another bribery game form that prevails in the literature. Implications for corrupt practices in relation to economic development are also discussed.

In this paper, we consider the basic framework of Beck and Maher (1986) and extend some of their results. Specifically, employing the symmetric Nash equilibrium concept, Beck and Maher (1986) showed that there is a fundamental isomorphism between competitive bidding and competitive bribery on the supply side of the transaction. Thus bribery may not result in any loss of efficiency in comparison with competitive bidding procedures.

With respect to reformists' concerns, the result is certainly more than welcome. [1] In fact, at a symmetric equilibrium, each firm is assumed to choose the optimal amount of bribe according to its gross profit (or cost) level using the same bribe-gross profit (cost) function as the other firms. Since this bribery function is assumed to be monotonically increasing with gross profit, or alternatively, monotonically decreasing with cost, the least cost firm will pay the largest bribe, hence it is awarded the prize. As a result, the bribery game generates a desirable outcome. That is, no loss of allocative efficiency is incurred in the awarding process.

At this point, a natural question is whether or not there exist asymmetric Nash equilibria in which bribery functions differ across firms. [2] If so, then the conclusion provided above may merely represent one of several possible situations, consequently, its value in explaining real-world problems is reduced. For example, there may exist an asymmetric equilibrium such that, for some cases, high cost firms pay larger bribes than low cost firms. Thus, instead of the least cost firm, some other firm is awarded the prize. In this paper, we consider the case in which n firms compete for a government procurement contract, each one knowing only its own production cost. Therefore, it is an n-person game with incomplete information. We then show that there exists a unique Nash equilibrium which is also symmetric. The reformists' position is hence upheld.

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[1] During the 60s, the massive prevalence of corruption in developing nations (especially in newly independent states) stimulated debate among political scientists over the effects of corruption upon economic development. Basically, moralists maintained that corruption is definitely detrimental while reformists argued that corruption provides some benefits to developing nations which in some cases may exceed the cost. One of the benefits is the allocative efficiency attained through competitive bribery procedures. For details, see Heidenheimer (1970).

[2] For an example in which both symmetric and asymmetric equilibria exist, see Nalebuff and Riley (1984).

Specifically, we assume n firms negotiate privately with a government official for a government procurement contract. The contract is awarded at a predetermined price P to the firm paying the largest bribe, this fact is common knowledge to all firms. We denote B_i the bribe paid by ith firm, $i = 1, \ldots, n$. Firms are assumed to know their own production cost levels c_i, $i = 1, \ldots, n$, but have incomplete information about rivals' cost levels. Furthermore, each firm assumes each rival's cost is independently drawn from a common distribution, which generates a common distribution of gross profit $F(z)$ over support $[\underline{z}, \bar{z}]$ where $z = P - c$ with c being a random variable denoting the representative rival's cost level; also $0 \leq \underline{z} < \bar{z}$. Two other assumptions are imposed: (i) $F(\cdot)$ is continuously differentiable, (ii) bribery functions $B_i(z)$ which associate z with B_i are strictly monotonically increasing with $B_i(\underline{z}) = \underline{z}$.

We now turn to firm 1's problem. Given its gross profit z_1, the firm attempts to choose the optimal amount of bribe to maximize the expected net profit,

$$\mathrm{E}[\phi(B_1)] = (z_1 - B_1)\ \Pr(B_j \leq B_1, \forall j = 1, \ldots, n). \tag{1}$$

Similarly, firm i attempts to choose B_i such that

$$\mathrm{E}[\phi(B_i)] = (z_i - B_i)\ \Pr(B_j \leq B_i, \forall j = 1, \ldots, n)$$

attains the maximum, given z_i. Under these specifications, all unsuccessful bribers will get refunds. Another specification in which bribers lose all the bribe whether successful or failed will be discussed later. [3] Using the common distribution $F(\cdot)$, the relationship $B_j = B_j(z_j)$, and the assumption that $\{z_j\}$ is a sequence of identically independently distributed random variables, we have

$$\mathrm{E}[\phi(B_1)] = (z_1 - B_1) \prod_{j=2}^{n} F\left[B_j^{-1}(B_1)\right].$$

The first order condition for optimal B_1 is then

$$(z_1 - B_1)\ \mathrm{d}\left\{ \prod_{j=2}^{n} F\left[B_j^{-1}(B_1)\right] \right\} \Big/ \mathrm{d}B_1 = \prod_{j=2}^{n} F\left[B_j^{-1}(B_1)\right].$$

Upon multiplying both sides by $(\mathrm{d}B_1/\mathrm{d}z)$ and noting the above equation holds for every z_1, we get

$$z\left(\left(\mathrm{d}\prod_{j=2}^{n} F\left[B_j^{-1}(B_1)\right] \Big/ \mathrm{d}B_1\right) \frac{\mathrm{d}B_1}{\mathrm{d}z}\right) = \left(B_1\left(\mathrm{d}\prod_{j=2}^{n} F\left[B_j^{-1}(B_1)\right] \Big/ \mathrm{d}B_1\right) + \prod_{j=2}^{n} F\left[B_j^{-1}(B_1)\right]\right) \frac{\mathrm{d}B_1}{\mathrm{d}z},$$

$$\forall z \in [\underline{z}, \bar{z}].$$

Thus,

$$\begin{aligned} B_1 \prod_{j=2}^{n} F\left[B_j^{-1}(B_1)\right] &= \int_{\underline{z}}^{z} t\left(\mathrm{d}\prod_{j=2}^{n} F\left[B_j^{-1}(B_1(t))\right] \Big/ \mathrm{d}t\right) \mathrm{d}t \\ &= z \prod_{j=2}^{n} F\left[B_j^{-1}(B_1)\right] - \int_{\underline{z}}^{z} \prod_{j=2}^{n} F\left[B_j^{-1}(B_1(t))\right] \mathrm{d}t. \end{aligned}$$

[3] In fact, this version may be more popular than that of Beck and Maher (1986) in the literature. See, for example, Macrae (1982).

As a result, we have

$$B_1(z) = z - \int_{\underline{z}}^{z} \prod_{j=2}^{n} F\left[B_j^{-1}(B_1(t))\right] \mathrm{d}t \Big/ \prod_{j=2}^{n} F\left[B_j^{-1}(B_1(z))\right], \quad \forall z \in [\underline{z}, \bar{z}].$$

Similarly,

$$B_i(z) = z - \int_{\underline{z}}^{z} \prod_{\substack{j=1\\ j\neq i}}^{n} F\left[B_j^{-1}(B_i(t))\right] \mathrm{d}t \Big/ \prod_{\substack{j=1\\ j\neq i}}^{n} F\left[B_j^{-1}(B_i(z))\right], \quad \forall z \in [\underline{z}, \bar{z}], \quad \forall i = 1,\dots,n. \tag{2}$$

We want to further investigate the relationship among the B_i's.

Specifically, we consider $B_1(z)$ in comparison with $B_2(z)$. Assume $B_1(z)$ and $B_2(z)$ are identical within $[\underline{z}, z_0]$ for some $z_0 < \bar{z}$ and that they intersect at least once over the interval $(z_0, \bar{z}]$. Let z^* denote the first intersection point. Then $B_1(z^*) = B_2(z^*)$ and $B_1(t) > B_2(t)$ [or $B_2(t) > B_1(t)$ in which case we will interchange the labels] whenever $t \in (z_0, z^*)$. Moreover, $B_1^{-1}(B_2(z^*)) = z^* = B_2^{-1}(B_1(z^*))$. Consequently, upon comparing $B_1(z^*)$ and $B_2(z^*)$ from eq. (2), we have

$$\int_{\underline{z}}^{z^*} F\left[B_2^{-1}(B_1(t))\right] \prod_{j=3}^{n} F\left[B_j^{-1}(B_1(t))\right] \mathrm{d}t = \int_{\underline{z}}^{z^*} F\left[B_1^{-1}(B_2(t))\right] \prod_{j=3}^{n} F\left[B_j^{-1}(B_2(t))\right] \mathrm{d}t. \tag{3}$$

On the other hand, when $t \in [\underline{z}, z_0]$, $B_1(t) = B_2(t)$ which implies $B_j^{-1}(B_1(t)) = B_j^{-1}(B_2(t))$, $\forall j$, and $B_1^{-1}(B_2(t)) = t = B_2^{-1}(B_1(t))$. If $t \in (z_0, z^*)$, then $B_1(t) > B_2(t)$. Thus $B_2^{-1}(B_1(t)) > t > B_1^{-1}(B_2(t))$, and $B_j^{-1}(B_1(t)) > B_j^{-1}(B_2(t))$, $\forall j$. As a result, the left-hand side of eq. (3) must be strictly greater than the right-hand side, which is a contradiction. In other words, when $B_1(z)$ and $B_2(z)$ are not everywhere identical, one function must be strictly greater than the other except in some initial interval where the two may be identical.

To proceed further, consider the way the competitive bribery game operates at Nash equilibria. In this case, bribery functions are common knowledge to all firms (which is a prerequisite for each firm to calculate its optimal amount of bribe). Moreover, each firm agrees to implement its bribery function after observing its gross profit level. Upon collecting all the bribes, the contract winner is determined, and refunding procedures start. Thus, it is necessary for all firms to have equal ex ante expected net profit, otherwise they will not agree upon the set of bribery functions, and a divergence from equilibrium will result. Now, from eq. (2), the ex ante expected net profit for firm i is

$$\mathrm{E}_z\mathrm{E}[\phi(B_i)] = \int_{\underline{z}}^{\bar{z}} \left(\int_{\underline{z}}^{s} \prod_{\substack{j=1\\ j\neq i}}^{n} F\left[B_j^{-1}(B_i(t))\right] \mathrm{d}t \right) \mathrm{d}F(s). \tag{4}$$

From previous results, if $B_1(z)$ and $B_2(z)$ are not identically equal, then $B_1(z) > B_2(z)$ [or $B_2(z) > B_1(z)$] except in some initial interval. Therefore,

$$\mathrm{E}_z\mathrm{E}[\phi(B_1)] > \mathrm{E}_z\mathrm{E}[\phi(B_2)] \left(\text{or } \mathrm{E}_z\mathrm{E}[\phi(B_2)] > \mathrm{E}_z\mathrm{E}[\phi(B_1)]\right).$$

That is, one firm has strictly greater ex ante expected net profit than the other. Again, we have contradictions. As a consequence, $B_1(z) = B_2(z)$ at Nash equilibrium. Following similar procedures, we can establish that $B_i(z) = B_j(z)$, $\forall i,\ j = 1,\dots,n$. This is summarized in Theorem 1.

Theorem 1. Assume each firm adopts the same distribution function $F(z)$. Within the competitive bribery game specified above, there exists a unique Nash equilibrium with the following bribery functions:

$$B_i(z) = z - \int_{\underline{z}}^{z} F^{n-1}(t)\,\mathrm{d}t \Big/ F^{n-1}(z), \quad \forall i = 1, \ldots, n. \tag{5}$$

If firms adopt different distribution functions of gross profit and if these functions have the same support, then the bribery functions for firms 1 and 2 are:

$$B_1(z) = z - \int_{\underline{z}}^{z} \prod_{j=2}^{n} F_j(t)\,\mathrm{d}t \Big/ \prod_{j=2}^{n} F_j(z),$$

$$B_2(z) = z - \int_{\underline{z}}^{z} F_1(t) \prod_{j=3}^{n} F_j(t)\,\mathrm{d}t \Big/ F_1(z) \prod_{j=3}^{n} F_j(z),$$

respectively, when the symmetric equilibrium exists. Since $B_1(z) = B_2(z)$ at the equilibrium, we have

$$\prod_{j=2}^{n} F_j(z) \Big/ \int_{\underline{z}}^{z} \prod_{j=2}^{n} F_j(t)\,\mathrm{d}t = F_1(z) \prod_{j=3}^{n} F_j(z) \Big/ \int_{\underline{z}}^{z} F_1(t) \prod_{j=3}^{n} F_j(t)\,\mathrm{d}t.$$

Hence,

$$\int_{\underline{z}}^{z} \prod_{j=2}^{n} F_j(t)\,\mathrm{d}t = c \int_{\underline{z}}^{z} F_1(t) \prod_{j=3}^{n} F_j(t)\,\mathrm{d}t \quad \text{for some constant } c,$$

which implies

$$\prod_{j=2}^{n} F_j(z) = cF_1(z) \prod_{j=3}^{n} F_j(z)$$

and hence $F_2(z) = cF_1(z)$. Because both $F_1(\cdot)$ and $F_2(\cdot)$ are cumulative density functions, $1 = F_2(\bar{z}) = cF_1(\bar{z}) = c$, thus $F_1(\cdot) = F_2(\cdot)$ identically. Following similar procedures, we have the following theorem.

Theorem 2. A necessary condition for symmetric Nash equilibria to exist in the above bribery game is that all firms adopt the same cumulative density function $F(z)$.

Upon combining the two theorems, Theorem 3 follows:

Theorem 3. The competitive bribery game has a unique Nash equilibrium that is symmetric if and only if all firms adopt the same cumulative density function over possible gross profit levels.

Some properties of the unique Nash equilibrium are in order. First, it is obvious that $B_i(z) > 0$. Furthermore, using integration by part, we can also show that $B_i(z) < z$ whenever $z > \underline{z}$. Thus firms always maintain positive profits unless their cost level is P. Another interesting property is that $B_i'(\underline{z}) = \frac{1}{2}$. That is, at the maximum cost level, an increase in the gross profit level will result in an equal division between the firm and the government official. The result is derived by applying L'Hospital rule to eq. (5), noting that $F'(\underline{z}) \neq 0$.

The above model assumes that unsuccessful bribers will get refunds. If, instead, firms always lose the bribes whether successful or failed, then the expected net profit for firm i is:

$$E[\pi(B_i)] = z_i \Pr(B_j \leq B_i, \quad \forall j = 1, \ldots, n) - B_i. \tag{6}$$

Using a similar approach and assuming that the bribery function satisfies $\tilde{B}_i(\underline{z}) = 0$, we derive the following bribery functions: [4]

$$\tilde{B}_i(z) = z \prod_{\substack{j=1 \\ j \neq i}}^{n} F\left[\tilde{B}_j^{-1}(\tilde{B}_i(z))\right] - \int_{\underline{z}}^{z} \prod_{\substack{j=1 \\ j \neq i}}^{n} F\left[\tilde{B}_j^{-1}(\tilde{B}_i(t))\right] \mathrm{d}t, \quad \forall z \in [\underline{z}, \bar{z}].$$

Again, it can be shown that Theorems 1–3 still hold. Therefore, we have a unique Nash equilibrium which is symmetric under the new model specification. In fact, $\tilde{B}_i(z) = F^{n-1}(z) B_i(z)$ at the equilibrium states. [5] Thus $\tilde{B}_i(z) < B_i(z)$ whenever $z < \bar{z}$. The result is certainly expected since the firm faces greater risk level in the new model. Thus, we have strengthened the conclusion of Beck and Maher (1986) since the symmetric equilibrium is the unique Nash equilibrium under two popular model specifications. That is, competitive bribery incurs no loss of allocative efficiency in comparison with competitive bidding procedures.

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[4] The assumption that $\tilde{B}_i(\underline{z}) = 0$ can be justified by the requirement of non-negative profit for each firm: since at $\underline{z}$, $F(\underline{z}) = 0$, hence $E[\pi(B_i)] \geq 0$ only when $\tilde{B}_i(\underline{z}) = 0$. In fact, if $\tilde{B}_i(\underline{z}) \neq \tilde{B}_j(\underline{z})$ for some i, j, then either $\tilde{B}_i^{-1}(\tilde{B}_j)$ or $\tilde{B}_j^{-1}(\tilde{B}_i)$ is not well-defined for some intervals; consequently, eq. (6) may be meaningless. Similar results hold if $B_i(\underline{z}) \neq B_j(\underline{z})$ for some i, j. However, in the first model, if we assume that $B_i(\underline{z}) = B_j(\underline{z})$ for every i, $j = 1, \ldots, n$, then $B_i(\underline{z}) = B_j(\underline{z}) = \underline{z}$ at the equilibrium.

[5] Upon applying integration by part, it can be shown that, at the unique Nash equilibrium, $B_i(z)$ is the conditional mean of the first order statistics $x = \max\{z_j, j = 1, \ldots, n\}$ conditional on the event $x \leq z$. On the other hand, $\tilde{B}_i(z)$ is the lower partial moment of x evaluated at z.